The Interbank Market Run and Creditor Runs[*]

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Abstract

This paper develops a general equilibrium model to study the interplay between the interbank market run (the run by banks on the interbank market, such as the (bilateral) Repo run) and creditor runs (runs by non-bank creditors on a bank) in a financial system. We first present a novel framework to endogenize the interbank market. In our framework, the illiquidity risk of banks is *endogenous*, originating in the insolvency risk. The role of the interbank market is to allow banks in the financial system with idiosyncratic solvency shocks to trade short-term funds to solve their illiquidity problem and thus mitigate the potential creditor runs. With this framework, we then show that the interbank market run and creditor runs interact and in fact reinforce each other, and the feedback can amplify a small aggregate shock into the joint event of "interbank market freezing" and "liquidity evaporating", which helps explain a systemic crisis.
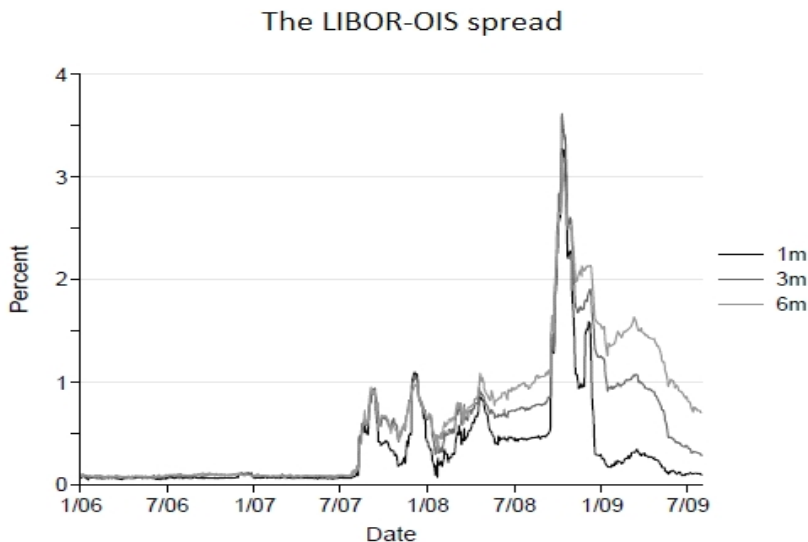
JEL classification: G01; G21; D83; D53

Keywords: Interbank market, creditor runs, coordination risk, global games, general equilibrium

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1 Introduction

A salient feature of the recent financial crisis of 2007-2009 was *systemic* creditor runs on financial institutions. Short-term creditors in the institutions rushed for the exit and liquidity evaporated abruptly. The modern-day bank runs that occurred in the shadow banks were a vivid illustration of the evaporation of liquidity.[1] Covitz et al. (2012) document that the runs on asset-backed commercial paper (ABCP) programs led to outstanding ABCP falling by \$400 billion (one-third of the existing amount) during the second half of 2007. Duygan-Bump et al. (2012) document that the runs on prime money market funds (MMFs) caused asset value to shrink from \$ 1300 billion to \$ 900 billion within one week after the collapse of Lehman Brothers. Notably, systemic creditor runs coincided with the "freezing" of the interbank market (Gorton and Metrick (2011b) and Covitz et al. (2012)). The LIBOR-OIS spread (a primary measure of interbank lending rates) increased to over 300 bps at the peak of the crisis, in contrast to the pre-crisis level of less than 10 bps (see Figure 1).[2]

Figure 1: The interbank lending rates

This paper proposes a novel rationale for interbank market lending and demonstrates the interdependence of the interbank market run and creditor runs. Our framework might also help in understanding the origin of crisis, amplification and contagion.

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[1] Runs on banks such as Northern Rock, Bear Stearns, Lehman Brothers, and others have been well recognized.

[2] Data is from British Banker's Association and Bloomberg.

In our framework, the role of the interbank market is to allow banks in the financial system with idiosyncratic fundamental shocks to trade short-term funds to solve their illiquidity problem and thus mitigate the potential creditor runs. The credit risk of debt in a financial institution can be decomposed into two parts: fundamental (insolvency) risk and coordination (illiquidity) risk. In our framework, the illiquidity risk is *endogenous*, originating in the insolvency risk. When the fundamental (insolvency) risk increases, the coordination problem among short-term creditors becomes more severe, who tend to run more likely on the institution, so the illiquidity risk also increases. Because institutions in the system have idiosyncratic fundamental shocks, the presence of an interbank market allows banks in the system to trade short-term funds to solve their illiquidity problems.

We develop a three-date model. At the initial date, each bank in the system makes its portfolio (liquidity) allocation: amount of cash holdings and investment in a long-term illiquid asset (with a higher expected return). At the intermediate date, banks realize their idiosyncratic fundamental shocks (asset quality). Higher asset quality of a bank means that the bank's long-term asset will realize the high cash flow with a greater probability at the final date. Creditors in a bank receive imperfect information about the type of their bank at the intermediate date, and need to make their rollover decisions. Creditors face a coordination problem between each other in rollover decisions.

We first solve the equilibrium at the intermediate date, that is, the equilibrium of a creditor-run game. In the benchmark case without an interbank market, we show that the probability of a creditor run for a bank is a decreasing function of the bank's *asset quality* and *cash holdings*. Intuitively, higher asset quality means that insolvency risk of the bank at the final date is lower, and thus creditors are more willing to roll over their lending. Higher cash holdings mean that the bank is more liquid and can withstand a larger proportion of creditors' interim withdrawals; hence, coordination risk (illiquidity risk) of the debt is lower. Therefore, cash holdings and asset quality are *substituting* in preventing a creditor run. In other words, a stronger bank, which has fewer creditors calling loans in equilibrium, needs less cash to prevent a creditor run. This gives rise to the role of an interbank market, where banks with heterogeneous fundamental shocks borrow and lend short-term funds among each other.

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In the case with an interbank market, we show that the probability of a creditor run for an individual bank depends on the bank's asset quality as well as the funding condition in the interbank market. The funding condition affects how much liquidity the bank can raise from the interbank market and thus how much total liquidity the bank can access. The funding condition of the interbank market, in turn, depends on other banks' status and their willingness to lend. Therefore, in the end, the probability of a creditor run for an individual bank depends on the bank's asset fundamentals as well as the status of the aggregate economy (more specifically, the asset quality distribution of banks in the system). Put slightly differently, with the *competitive* interbank market, the interbank market rate and the rollover decision of creditors in an individual bank are *jointly* determined in equilibrium, which in turn is dependent on the status of the aggregate economy.

Then, we move to solve the equilibrium at the initial date. We consider both the constrained second-best equilibrium and the competitive equilibrium (the *laissez-faire* equilibrium). In the constrained second-best equilibrium, the social planner aims to maximize the aggregate value of all banks in the economy by setting an optimal amount of cash holdings for banks. The social planner faces the following tradeoff: a higher amount of cash holdings in banks results in more banks in the system surviving at the intermediate date, but also less investment in long-term assets (with a higher return) in the economy. The tradeoff leads to a unique optimal amount of cash holdings. In the competitive equilibrium, banks decide their ex ante cash holdings based on individual rationality. Concretely, every bank takes the interbank rate *as given*. Thus, banks trade off two investment opportunities at the initial date: investing cash in the long-term asset immediately and saving cash to lend in the interbank market later. We find a unique competitive equilibrium. In comparing these two equilibria, we find that the second best efficiency, in general, cannot be implemented by the competitive equilibrium. There is an over-investment of liquidity in the competitive equilibrium.

Finally, we study our model under an aggregate shock, which might help in understanding the origin of crisis, amplification and contagion. A crisis occurs when an adverse *aggregate shock* on asset quality hits the economy. Concretely, when some banks (perhaps high-quality banks) realize worse asset quality than they expect, they need to hoard more liquidity in order to mitigate their illiquidity problems. This decreases the supply of short-term funds in the interbank market, with the result that the interbank rate goes up. As the interbank rate increases, *ceteris paribus*, an

individual bank can raise less liquidity in the interbank market and thus becomes less capable of meeting its creditors' early withdrawals. This triggers higher *coordination risk* among creditors in rollover, who run more often. In anticipation of this, even the banks that do not suffer a shock on asset quality increase their liquidity hoardings to protect themselves, leading to an increase in the aggregate demand for liquidity, and consequently an even higher interbank rate, more severe runs on banks, and so on. In short, an adverse aggregate shock triggers a reinforcing spiral of a rise in the interbank rate, a more severe coordination problem among creditors in debt rollover decisions, and greater liquidity hoarding demand of banks, resulting in the phenomenon of "liquidity evaporation" simultaneous with "interbank market freezing".

Our paper demonstrates financial contagions that occur through the channel of institutions fishing short-term funding from the same pool. This might help explain some important phenomena in the recent crisis. For example, the crisis originated in the US, where there had been an asset fundamental shock - the subprime mortgage crisis; however, the first bank that suffered bank runs was Northern Rock, a UK bank. Also, when Northern Rock was experiencing runs in July 2007, there had been no (large-scale) *fire sales* or liquidations yet around the world. We emphasize the very reason that Northern Rock and the US institutions shared the common short-term funding market.

As for the regulatory implications, our model conveys two messages. First, the ex ante liquidity regulation on financial institutions improves social welfare, which can solve the problem of inefficient supply of liquidity of individual institutions. Second, when a crisis occurs, an important ex post intervention policy is to inject liquidity into the financial system, which is crucial to break the vicious feedback cycle. This justifies the traditional as well as less traditional policy actions by the Federal Reserve and other major central banks in the recent crisis.[3] For example, in addition to cutting interest rates to practically zero, the Federal Reserve created emergency liquidity facilities (e.g., the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility). In fact, the debate on whether central banks should provide emergency liquidity assistance went well before the recent crisis. Goodfriend and King (1988) (see also Bordo (1990), Kaufman (1991) and Schwartz (1992)) remark that banking policy was necessary at a time when financial markets were underdeveloped; however, "with sophisticated interbank markets, banking policy has become

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[3] Allen et al. (2009) and Freixas et al. (2011) focus on studying ex post central bank interventions.

redundant" – in other words, they argue that with a well-functioning interbank market, a solvent institution cannot be illiquid.[4] Our work highlights that coordination failures and self-fulfilling beliefs under market frictions (imperfect information) can play a large role in a crisis. Through the actions and reactions within a bank and between banks, a small shock on the solvency issue in the system can trigger the endogenous cycle of illiquidity and insolvency of banks.

Empirical evidence. Recent literature documents the empirical evidence on runs in the financial system. Copeland et al. (2011), Gorton and Metrick (2010, 2011a, 2011b), and Krishnamurthy et al. (2012) provide evidence on the repo run. The run on prime money market funds (Duygan-Bump et al. (2012)) and the run in the ABCP market (Kacperczyk and Schnabl (2010), Acharya and Schnabl (2010) and Covitz et al. (2012)) have also been documented vividly.

In a series of papers, Gorton and Metrick (2010, 2011a, 2011b) have argued that the "run on repo" played a key role in the collapse of the shadow banking system. Krishnamurthy et al. (2012), on the other hand, find that the contraction in repo funding flows from non-bank lenders to shadow banks was small; the magnitude was relatively insignificant compared with the contraction in ABCP. Our theoretical work provides a unified framework to explain the evidence in these papers. Our model implies the importance of distinguishing two types of repo lending: non-bank to dealer repo lending (largely tri-party) and interbank repo lending between dealers (largely bilateral),[5] and demonstrates the interplay between the interbank market run (such as the bilateral Repo run) and creditor runs (such as runs through ABCP and tri-party repo).

As for determinants of creditor runs, Covitz et al. (2012) document that at the program (firm) level, runs were more likely on ABCP programs with weak fundamentals. Both Gorton and Metrick (2011b) and Covitz et al. (2012) find that the probability of runs was strongly correlated with the macro-economic shocks – the ABX index, which proxies for asset fundamentals, and the LIBOR-OIS spread, which measures the interbank lending rates. Our model explains the above evidence. Based on our model, two key factors determining the likelihood of an individual-bank run are a bank's asset fundamentals and the funding condition in the interbank market, while the funding condition is in turn determined by the aggregate risk of asset quality in the economy.

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[4]See Rochet and Vives (2004) and Freixas and Rochet (2008) for a comprehensive discussion on this debate.

[5]According to Krishnamurthy et al. (2012), the repo run studied in Gorton and Metrick (2011b) is mainly on the repo lending between banks.

Acharya and Merrouche (2013) document evidence of precautionary liquidity-hoarding of banks during the recent crisis. They find that the liquidity demand of large settlement banks in the UK experienced a sharp increase in the period immediately following the day on which money markets froze, igniting the crisis. They further provide evidence that the liquidity hoarding was due to precautionary motives rather than counterparty risk effects, at least in the early phase of the crisis. These findings are consistent with our model.

In the case study on Northern Rock, Shin (2009) highlights contagions in the recent crisis. According to Shin (2009), the failure of Northern Rock was probably not because it suffered a loss of its deposits or investment in banks in the US, as Northern Rock had "virtually no subprime lending". Instead, Shin (2009) emphasizes that Northern Rock fished from the same pool of short-term wholesale funding as did the US institutions.

The rest of the paper is organized as follows. Section 2 sets up the model. Sections 3 and 4 present the equilibria. Section 5 studies the model under an aggregate shock. Section 6 discusses related literature. Section 7 concludes.

2 The model

We present the model in this section. We discuss banks, the interbank market, and bank runs in order. All agents are risk-neutral.
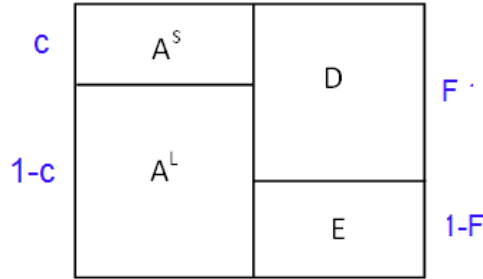
2.1 Banks

The model has three dates: T_0, T_1, and T_2, and there is no time discount for simplicity. There is a continuum of banks with measure 1, indexed by $i \in [0, 1]$. Ex ante, at T_0, these banks are identical. Each bank has one unit of cash (capital), which is financed by the owner of the bank (hereafter, equityholder or bankowner) as well as a continuum of short-term creditors of total measure F, each contributing 1 unit of cash.[6] That is, the liability side of the balance sheet of a bank at T_0 includes

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[6]The short-term debt in our model may play a role of disciplining (Calomiris and Kahn (1991), Diamond and Rajan (2001)). For example, without the threat of the short-term debtholder run, the owner of a bank can take an (off-equilibrium) action which makes the bank asset being more risky with a negative NPV. Kacperczyk and Schnabl (2010) provide indirect evidence – many investors "purchase newly issued commercial paper from the same issuer once their holdings of commercial paper mature."

the debt of total face value F and the equity value $1 - F$, where $0 < F < 1$. We can think of each bank as a regional or sectoral bank with its own investors (creditors) pool. A creditor's reserve value (the opportunity cost) of lending is R_0 (per unit of cash), where $R_0 \geq 1$; alternatively, we can assume that a creditor can access a constant-return-to-scale technology with productivity (TFP) R_0.

At T_0, a bank needs to make its investment portfolio allocation: amount of cash holdings and investment in a long-term asset (technology). Each bank has access to a long-term risky asset (technology) with stochastic payoffs at T_2. The unit cost of a long-term asset at T_0 is 1. If a long-term asset is liquidated at T_1, it realizes zero liquidation value (which will be elaborated on further). Denote the investment decision of a bank at T_0 by $(c, 1 - c)$, where c is the cash holdings and $1 - c$ is the investment in the long-term asset.[7] A bank's balance sheet at T_0 is represented by Figure 2, where A^S and A^L are the short-term liquid asset and the long-term illiquid asset, respectively.
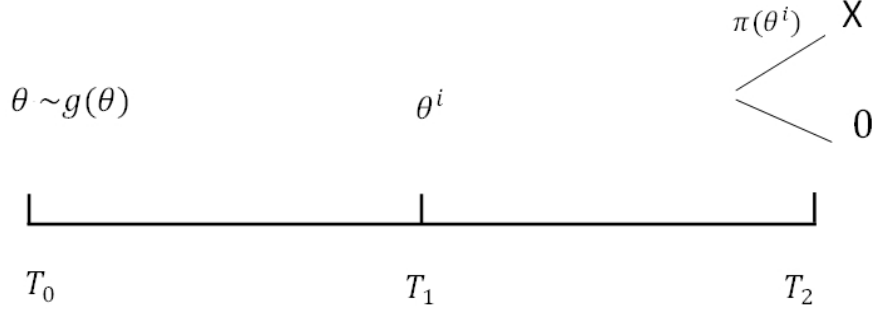

Figure 2: Balance sheet of banks at T_0

The uncertainty on the payoff of a bank's long-term asset will be resolved gradually, as shown in Figure 3. The uncertainty at T_0 is characterized by random variable θ, interpreted as *asset quality*. θ has smooth density $g(\theta)$ and cumulative distribution function $G(\theta)$ in the support $[\underline{\theta}, \overline{\theta}]$, where $-\infty \leq \underline{\theta} < \overline{\theta} \leq +\infty$.[8] The mean of θ is μ and its standard deviation is σ. At T_1, the uncertainty about each individual bank's asset quality is resolved. Specifically, bank i realizes its asset quality θ^i, where θ^i is *independently* drawn from the *identical* probability distribution $\theta \sim g(\theta)$. That is, banks are identical ex ante but heterogeneous (with idiosyncratic shocks) at T_1. At T_2, one of the

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[7] The investment decision $(c, 1 - c)$ is not publicly observed unless there is regulation on it, as in Bhattacharya and Gale (1987) as well as Acharya et al. (2013).

[8] The real line is both an open and a closed set. We can write it as $[-\infty, +\infty]$.

two cash flows, $\{0, X\}$, will be realized per unit of a long-term asset. The probability of realizing X for an asset of quality θ^i is $\pi(\theta^i)$, where $\pi(\cdot)$ is a continuous and increasing one-to-one function, $\pi : [\underline{\theta}, \overline{\theta}] \to [\underline{\pi}, \overline{\pi}]$, with $0 \le \underline{\pi} < \overline{\pi} \le 1$. We also assume that $\int_{\underline{\theta}}^{\overline{\theta}} [X \cdot \pi(\theta)] \, g(\theta) d\theta > 1$, which means that investing in a long-term asset is profitable ex ante at T_0.

Figure 3: The uncertainty on the payoff of a long-term asset

Further, we assume that the asset payoff realizations at T_2 across banks are correlated (for, e.g., the systemic risk of the economy as in Holmstrom and Tirole (1997)). The specific setup for the correlated asset payoffs is provided in Appendix. Basically, under correlated asset payoffs, the risk of the debt of a higher-quality bank cannot be reduced when it acquires some assets from lower-quality banks. This setup is not only realistic but also makes our analysis convenient.

The owner of a bank gets informed, at T_1, of the quality of his bank's long-term asset as well as the quality of assets of other banks (to be elaborated on further). It is realistic to assume that bankers, as insiders, have information about each other's status. But a creditor in a bank only receives a signal at T_1 regarding the quality of his own bank's long-term asset. Specifically, the signal for creditor j in bank i (about asset quality θ^i) at T_1 is $\theta^{ij} = \theta^i + \delta \epsilon^j$, where $\delta > 0$ is constant, and the individual specific noise ϵ^j is distributed according to the smooth symmetric density $h(\cdot)$ with mean 0 (writing its c.d.f. $H(\cdot)$). ϵ^j is i.i.d. across its creditors in a bank, and each is independent of θ^i. Similar to creditors, outsiders including a court cannot observe or verify the asset quality of a bank at T_1, as in the incomplete contract literature. This setup or the similar is employed in a large literature including Diamond and Dybvig (1983) and Bhattacharya and Gale (1987): the realization of the idiosyncratic shock is unverifiable.

Creditors of a bank are offered demand-deposit-like contracts: if a creditor calls his loan at T_1, his claim is the par (face) value 1; if, instead, he decides to roll over his loan until T_2, the *notional*

(promised) value of his claim is R, where R is the gross interest rate, to be endogenized.[9]

Example For tractability, we can use the following distributions and function specifications throughout the paper. The (prior) distribution of θ is assumed to be normal. Specifically, $\theta \sim N(\mu, \sigma^2)$; that is, $g(\theta) = \frac{1}{\sigma}\phi(\frac{\theta-\mu}{\sigma})$, where $\phi(\cdot)$ stands for the p.d.f. of the standard normal distribution, and $\underline{\theta} = -\infty$ and $\overline{\theta} = +\infty$. The c.d.f of θ is thus $G(\theta) = \Phi(\frac{\theta-\mu}{\sigma})$, where $\Phi(\cdot)$ is the c.d.f. for the standard normal distribution. Further, we can assume that $\pi(\theta) = \underline{\pi} + (\overline{\pi} - \underline{\pi})\Phi(\frac{\theta-\mu_\pi}{\sigma_\pi})$, where μ_π and σ_π are parameters. If $\mu_\pi = \mu$ and $\sigma_\pi = \sigma$, it is easy to show that $\pi(\cdot)$, as a function of θ, is *uniformly* distributed with $\pi(\theta) \sim U[\underline{\pi}, \overline{\pi}]$; that is, the probability of a bank's realizing X of its long term asset at T_2 perceived at T_0 is uniformly distributed within $[\underline{\pi}, \overline{\pi}]$. The distribution of signal noise ϵ^j is assumed to be the standard normal: $\epsilon^j \sim N(0, 1)$, that is, $h(\cdot) = \phi(\cdot)$ and $H(\cdot) = \Phi(\cdot)$.

2.2 Interbank market

There is an interbank credit market open at T_1, where banks can borrow and lend short-term funding between each other. For instance, the interbank market can be the (bilateral) repo market. Denote the total amount of cash available to a bank at T_1 by c^T, which includes the bank's own cash holding (c) and the net borrowing from the interbank market. Without loss of generality, creditors in a bank cannot observe the liquidity status (c^T) of the bank.[10] This is because, as will be shown, banks trading liquidity in the interbank market and creditors making rollover decisions take place *simultaneously*.

Two comments are in order:

First, in our general-equilibrium model, when a bank raises liquidity at T_1, the liquidity must be from other banks *in the system*. Interbank borrowing means that a bank obtains some cash

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[9]Without loss of generality, we normalize the interim nominal claim at T_1 to be 1. What matters for the model is the interest rate between T_1 and T_2, the R.

[10]This assumption itself is realistic. In the recent crisis, for example, a big problem was that outsider investors had no clue about the balance sheet compositions and the liquidity status of banks. From the theoretical perspective, we can consider that creditors also receive noisy signals about their bank's liquidity status. This is equivalent to creditors having another signal about the asset quality of their bank (because liquidity status is a one-to-one function of asset quality in a bank). When creditors' private information about asset quality is precise enough, this additional piece of information does not change the model results qualitatively.

at T_1 from other banks but gives up some of its cash flow (from its long-term project) at T_2 to repay other banks. That is, there is an exchange between T_1-cash flow and T_2-cash flow. Like Bhattacharya and Gale (1987), the interbank rate is defined as the ratio of the expected T_2-cash flow (repayment) to the T_1-cash flow (borrowing).

The effect of raising liquidity by interbank borrowing (against assets as collateral) is similar to the effect of asset selling/liquidation. Conceptually, however, there is an important difference between "borrowing/lending" and "selling/buying" (see, e.g., Acharya and Viswanathan (2011) for a discussion). Raising liquidity by selling means that transferring ownership and control rights of the asset is necessary, while borrowing (*against the cash flows at T_2 generated by the asset as collateral*) means that the asset is still operated by the borrower (i.e., the owner of the asset). In our model, to avoid *double counting* the liquidity-raising ability of an asset and focus on studying interbank lending, we assume that the liquidation/selling value of a bank's long-term asset at T_1 is zero. This can be justified by the fact that bank assets can be so *specific* or not "mature" enough yet at T_1 (Hart and Moore (1994)) that no other banks can operate them efficiently (for instance, every bank is a regional or sectoral bank), but once the cash flow at T_2 is generated it is collectable by lenders, so a bank has to raise liquidity by means of borrowing. Bhattacharya and Gale (1987), Allen and Gale (1998), and others also assume that liquidation is impossible.[11]

Second, as in Rochet and Vives (2004), we assume that the interbank market is informationally efficient, in the sense that lender banks have information about the asset quality of the borrower bank. Hence, the amount of liquidity that a bank can raise in the interbank market depends on the bank's asset quality.

2.3 Bank runs

If a bank has a number of creditors greater than c^T to decline to roll over their lending, the bank cannot fulfill its debt obligation, and consequently it fails – a creditor run.[12]

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[11] Empirically, when Northern Rock was experiencing runs in July 2007, there had been no (large-scale) fire sales or liquidations yet around the world. The failure of Northern Rock was instead triggered by the suddenly tightening funding condition of the interbank (wholesale funding) market (see, e.g., Shin (2009)).

[12] The liquidation value of any unit of the long-term asset is 0. The bank fails as long as the liquidation starts. In equilibrium, there are no partial liquidations.

Following the work of Rochet and Vives (2004) and Morris and Shin (2009), we use a simplified payoff structure of the creditor run game. Specifically, as in their work, we can assume that each creditor in a bank is an institutional investor (a fund), which is run by its fund manager. A fund manager has the following compensation scheme. If the fund manager calls on the fund's investment at T_1, the fund manager's payoff is a constant w_0, or the face value 1 multiplied by proportion w_0. This can be because, as shown later, calling loans at T_1 makes a creditor either fully recover the face value of investment 1 or suffer a small loss. w_0 can also be interpreted as the fund manager's monetary compensation with the non-pecuniary penalties deducted (e.g., his reputation losses due to early calls) as in Rochet and Vives (2004). If, instead, the fund manager holds, his payoff is the fund's return multiplied by γ. The ratio $\frac{w_0}{\gamma}$ is called the "outside option ratio" in Morris and Shin (2009). Table 1 shows the simplified payoff structure (for a fund manager) of the creditor run game, where the bank has asset quality θ at T_1.

	Calling no greater than c^T (bank survives)	Calling greater than c^T (bank fails)
Hold	$\begin{cases} \gamma R & (\text{prob } \pi(\theta)) \\ 0 & (\text{prob } 1 - \pi(\theta)) \end{cases}$ [13]	0
Call	w_0	w_0

Table 1: Creditor-run payoff structure

By introducing a third party, fund managers, we have a discrete-states payoff structure of the creditor-run game. This structure has the advantage of being simple and avoiding technique complications, and at the same time sufficing to capture the key feature of a creditor-run game — the (global) strategic-complementarities payoff structure: If the proportion of creditors in a bank higher than $\frac{c^T}{F}$ calls the loans, the optimal strategy for an individual creditor is to also 'call'. If, however, the proportion of the creditors calling the loans is less than $\frac{c^T}{F}$, the optimal strategy for an individual creditor is likely to also 'hold'.

In fact, if we substitute $w_0 = 1$ and $\gamma = 1$ into Table 1, the payoff structure is about the

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[13]We will verify that only when the long-term asset of a bank pays X at T_2, is the interest R realized. In other words, a fund manager obtains γR with probability $\pi(\theta)$ conditional on the bank surviving to T_2.

payoff to a bank creditor without assuming the involvement of a fund manager. In this case, the simplified payoff structure is a *close approximation* of the full continuous-states payoff structure as in Goldstein and Pauzner (2005) as well as Dasgupta (2004) and Liu and Mello (2012) (see Eq. (9)). Throughout the paper, we can assume that $\frac{w_0}{\gamma} = 1$ (with a fund manager), which has the same effect in *rollover decisions* as the assumption that in Table 1 $w_0 = 1$ and $\gamma = 1$ and there is no involvement of a fund manager (see Eq. (1')). In addition, we can assume that $w_0 \to 0$ and $\gamma \to 0$, which means that the compensation to the fund manager is negligible relative to the total payoff to the creditor.

2.4 Timeline

Without loss of generality, we assume that at T_2 the principal and interest payment to bank (existing) creditors, who are offered demand-deposit-like contracts, is senior to the repayment to counterparty banks for the interbank borrowing.[14]

The timeline is the following:

At T_0: 1) The owner (equityholder) of a bank attracts deposits by offering a nominal (gross) interest rate R. The owner has capital $1 - F$ and the total available capital (cash) from the potential depositors for one bank is F.

2) A bank and its depositors (creditors) sign the demand-deposit-like contracts.

3) A bank makes its investment decision $(c, 1 - c)$.

At T_1: 1) The owner of a bank gets informed of the asset quality of his bank as well as the asset quality of other banks. Banks borrow and lend short-term funding in the interbank lending market. The interbank rates are determined by the market equilibrium.

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[14]This assumption is purely for tractability. It is to make the cash flow pattern for each claimant simple, so that we can focus on the core of our mechanism. In fact, if interbank lenders are senior to non-bank creditors, the bank can default to a non-bank creditor at T_2 (even if the bank is liquid and thus has no defaults at T_1), in which case the cash flow pattern for a non-bank creditor is very complicated and his participation condition (IR) is hard to make tractable although the model result does not change qualitatively.

A creditor (fund manager) in a bank receives a signal regarding the asset quality of his bank. Based on his signal, he decides whether to roll over his credit to the bank. All creditors make the rollover decisions simultaneously.

2) If a sufficient number of creditors in a bank declines to roll over, the creditor run occurs and the bank fails.

At T_2: 1) If a bank survives to T_2, the payoff of its long-term asset is realized.

2) For a bank that has borrowed in the interbank market, its depositors, counterparty banks, and equityholder get payoffs, in order. A bank that has lent in the interbank market receives the repayment from its counterparty banks first and then pays its creditors and equityholder, in order.
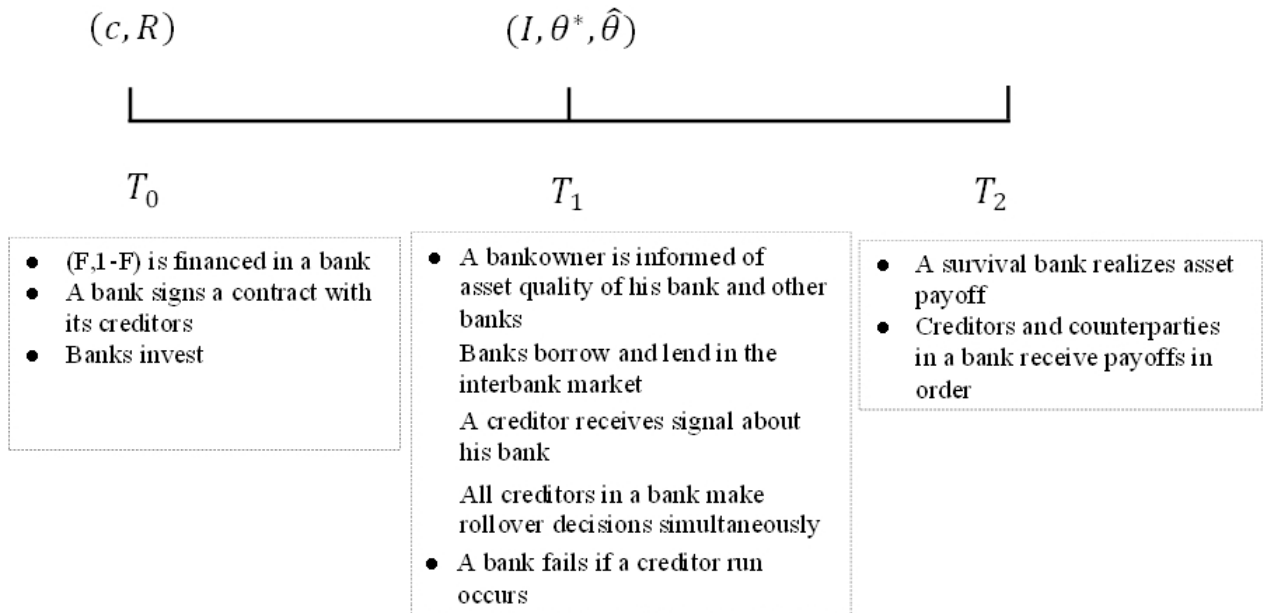
Figure 4 summarizes the timeline.

(c, R) $(I, \theta^*, \hat{\theta})$

T_0 T_1 T_2

- (F,1-F) is financed in a bank
- A bank signs a contract with its creditors
- Banks invest

- A bankowner is informed of asset quality of his bank and other banks
  Banks borrow and lend in the interbank market
  A creditor receives signal about his bank
  All creditors in a bank make rollover decisions simultaneously
- A bank fails if a creditor run occurs

- A survival bank realizes asset payoff
- Creditors and counterparties in a bank receive payoffs in order

Figure 4: Timeline

3   Equilibrium at T_1 - endogenous interbank market

We conduct analysis by backward induction, from T_1 to T_0. In this section, for given c and R, which are set at T_0, we work out the equilibrium at T_1. The main purpose of this section is to

endogenize the interbank market.

At T_1, creditors in a bank need to make their rollover decisions, that is, they play a creditor-run game. We are interested in the equilibrium where every creditor uses a threshold (monotone) strategy. The strategy is given by

$$\theta^{ij} \longmapsto \begin{cases} Call & \theta^{ij} < \theta^* \\ Hold & \theta^{ij} \geq \theta^* \end{cases},$$

where θ^{ij} is the information of creditor j in bank i and θ^* the threshold.[15] Note that a creditor in a bank does not have bank-specific information about his bank, so the threshold θ^* is not bank-specific. That is, creditors in all banks use a common strategy. Also, the prior distribution of θ, the density $g(\theta)$, is equivalent to *public information* about the asset quality for a creditor (see also Morris and Shin (2003)). So a creditor's strategy θ^* depends on public information. We assume that $0 << \frac{\delta}{\sigma} << +\infty$, that is, both private and public information is valuable.

We first consider the benchmark equilibrium when there is no interbank lending market, and then consider the equilibrium when there is an interbank market.

3.1 Benchmark equilibrium at T_1 if there is no interbank lending market

In this case, we assume that banks are in autarky and there is no interbank lending market at T_1. So, $c^T = c$.

We solve the creditor-run game equilibrium at T_1 for given c and R. We define another threshold $\widehat{\theta}^i$, which is the equilibrium bank failure threshold for *an individual bank*, that is, if and only if the bank's realized fundamental value θ is below $\widehat{\theta}^i$, does the bank fail at T_1. Here, we use superscript 'i' in notation $\widehat{\theta}^i$ to highlight the fact that creditors in each *individual* bank are with 'local thinking' – they only consider the position of their own bank and do not necessarily have a global view of the banking system.

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[15] In the finance literature on applications of global games, the threshold (monotone) equilibrium is the primary interest. For example, Morris and Shin (2004b, 2009) consider only threshold (monotone) equilibria. As in Goldstein and Pauzner (2005), if an upper dominance region is assumed, the "bad" equilibrium is ruled out for sufficiently strong bank fundamentals.

For a given $\widehat{\theta}^i$, we consider the position of a creditor whose decision proxy is his fund manager. The *marginal* creditor (fund manager) in a bank who receives signal θ^* is indifferent to holding or calling; so we have

$$\int_{\underline{\theta}}^{\widehat{\theta}^i} 0 \cdot h_g(\theta|\theta^*)d\theta + \int_{\widehat{\theta}^i}^{\overline{\theta}} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta|\theta^*)d\theta = w_0, \tag{1}$$

where $h_g(\theta|\theta^*)$ is the posterior (conditional) density with the prior being $g(\cdot)$. The right hand side of (1) is the payoff of calling. The left-hand side of (1) expresses the payoff for the marginal creditor (fund manager) when he decides to roll over: If $\theta < \widehat{\theta}^i$, the bank cannot survive to T_2 and he gets nothing, which is the first term; conditional on the bank surviving to T_2, his expected payoff is $(\gamma R) \cdot \pi(\theta)$ for a given realization of θ, the second term. Equation (1) can be rewritten as

$$\int_{\underline{\theta}}^{\widehat{\theta}^i} 0 \cdot h_g(\theta|\theta^*)d\theta + \int_{\widehat{\theta}^i}^{\overline{\theta}} (R \cdot \pi(\theta)) \cdot h_g(\theta|\theta^*)d\theta = \frac{w_0}{\gamma}. \tag{1'}$$

From (1'), when $\frac{w_0}{\gamma} = 1$ the rollover decision is identical to the case that $w_0 = 1$ and $\gamma = 1$ in Table 1 and a creditor makes his rollover decision directly without involving a fund manager.

For a given θ^*, we consider the position of a bank. For a bank with fundamental value being θ, the proportion of its creditors calling is $H(\frac{\theta^*-\theta}{\delta})$. Hence, the bank with marginal fundamental value $\widehat{\theta}^i$ has a proportion $H(\frac{\theta^*-\widehat{\theta}^i}{\delta})$ of its creditors calling. By the definition of $\widehat{\theta}^i$ and the nature of creditor runs, we have

$$c = F \cdot H(\frac{\theta^* - \widehat{\theta}^i}{\delta}). \tag{2}$$

The system of equations (1) and (2) determines the creditor-run equilibrium. We have the following result.

Lemma 1 *Without an interbank market, the creditor-run equilibrium at T_1 is characterized by the pair $(\widehat{\theta}^i, \theta^*)$, which solves the system of equations (1)-(2) for given c and R. The creditor-run game has a unique (stable) equilibrium under some parametric values. We have the comparative statics: $\frac{\partial \theta^*}{\partial c} < 0$ and $\frac{\partial \theta^*}{\partial R} < 0$.*[16]

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[16]In a "stable" equilibrium, the best response function (of an individual player to its peers) intersects the 45^0 line at a slope of less than 1. It is known in the literature that even if the global game has multiple equilibria, comparative statics apply for a given equilibrium (see, e.g., Vives (2005, 2013)).

Proof: see Appendix.

The intuition behind the comparative statics in Lemma 1 is as follows. If a bank has more cash holdings, the bank can withstand withdrawals by more creditors and is less fragile, thereby lowering the threshold for coordination among creditors to not run. In other words, a higher c means that the chance of the accident of miscoordination among creditors at T_1 is lower; consequently, creditors are more comfortable with staying until T_2 and thus set a lower running threshold θ^*. A higher R means that creditors have a higher stake in the bank at T_2 and thus have lower incentives to run.

It is instructive to examine the creditor-run equilibrium under the limit $\delta \to 0$ for a given σ. By (2), we have $\widehat{\theta}^i = \theta^* - \delta\Phi^{-1}(\frac{c}{F})$. So we can combine (1) and (2):

$$\int_{\theta=\theta^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} (\gamma R) \cdot \pi(\theta) \cdot h_g(\theta|\theta^*)d\theta = w_0$$

Under the limit $\delta \to 0$ for a given σ, it is easy to show that the above equation can be transformed to

$$\underbrace{\frac{c}{F}}_{\text{Coordination (illiquidity) risk}} \cdot \underbrace{R \cdot \pi(\theta^*)}_{\text{Fundermental (insolvency) risk}} = \frac{w_0}{\gamma}. \tag{3}$$

(3) becomes very intuitive: the term $\frac{c}{F}$ measures coordination (illiquidity) risk while $R \cdot \pi(\theta^*)$ corresponds to fundamental (insolvency) risk. The uniqueness of the solution of θ^* is straightforward (under proper parametric values of c, F and R). The economic intuition behind the derivation of (3) is the following. Under the limit $\delta \to 0$, fundamental uncertainty disappears, i.e., $\theta \to \theta^*$. However, strategic uncertainty does not. From the marginal creditor's perspective, the proportion of creditors calling loans is uniformly distributed within $[0,1]$. So the probability that the proportion of creditors calling loans less than $\frac{c}{F}$ is exactly $\frac{c}{F}$.

From (3), θ^* is clearly decreasing in c and R, which confirms Lemma 1. Because $\delta \to 0$, we also have that $\widehat{\theta}^i = \theta^*$ and that $\widehat{\theta}^i$ is a decreasing function of c. Default risk of debt in a financial institution can be decomposed into two components: illiquidity risk and insolvency risk. (3) clearly shows that liquidity holdings, c, in our model affect *illiquidity risk* (or the bank survival rate at T_1), rather than *insolvency risk* of the debt. When c is low, even if the bank's fundamental value θ is sound, the bank may fail at T_1 and thus its debt defaults because the bank is illiquid, rather

than insolvent. This result crucially differentiates our model from the existing insights of Allen and Gale (2000) and Dasgupta (2004), where the failure of the debtor bank impacts the *capital* and thus *solvency* of the creditor bank. In fact, as shown later, in our model the liquid asset, c, has no impact on the bank's solvency or the riskiness of its debt at T_2. Whether the bank defaults or not at T_2 is completely determined by whether the bank's long-term asset pays 0 or X.

3.2   Equilibrium at T_1 if there is an interbank lending market

If there is no interbank lending market at T_1, the cash holdings of a bank are either not enough or wasted at T_1. Now we consider that there is an interbank lending market open at T_1, so banks can borrow and lend funds among each other.

The equilibrium at T_1 is characterized by the triplet $(\theta^*, \widehat{\theta}, I)$ for given c and R, where θ^* is the calling threshold of a creditor in a bank, $\widehat{\theta}$ denotes the marginal bank in the system that survives at T_1, and I is the *risk-adjusted* gross interbank market rate. The equilibrium comprises two elements: the creditor run equilibrium for an *individual* bank and the interbank market equilibrium.

The creditor run equilibrium for an individual bank   This equilibrium is given by the system of equations (4) and (5):

$$\int_{\underline{\theta}}^{\widehat{\theta}^i} 0 \cdot h_g(\theta|\theta^*)d\theta + \int_{\widehat{\theta}^i}^{\overline{\theta}} (R \cdot \pi(\theta)) \cdot h_g(\theta|\theta^*)d\theta = \frac{w_0}{\gamma} \tag{4}$$

$$\widehat{\theta}^i = \max(\underline{\theta}, \theta^T) \tag{5}$$

$$\text{where } \theta^T = \min \left\{ \theta \mid \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)\right] \pi(\theta)}{I} + c \geq F \cdot H(\frac{\theta^* - \theta}{\delta}) \right\} \tag{5a}$$

(4)-(5) parallels (1)-(2). The difference is that with an interbank market, an individual bank can borrow liquidity from it, so we have (5), in comparison with (2). In fact, if the borrowing is impossible (i.e., $I = +\infty$), (5) is identical to (2). Importantly, under the *competitive* interbank market, every bank is a price-taker and takes the interbank rate I as given. The liquidity status of the bank depends on its own liquidity holdings (c) and the short-term funding it can raise from the interbank market. (5) gives the condition under which the bank will fail for a given interbank rate I.

We explain (5). When the bank is of asset quality θ, a number of $F \cdot H(\frac{\theta^* - \theta}{\delta})$ creditors calls

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on it at T_1, and the remaining creditors with a number of $F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)$ stay until T_2. Hence, the expected payoff that will accrue to the bank's equityholder, after paying its staying creditors, is $\left[(1 - c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)\right] \pi(\theta)$ at T_2. Figure 5 shows the balance sheet position for the bank at T_1. So the bank can borrow a maximum amount of $\frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)\right] \pi(\theta)}{I}$ in the interbank market at T_1. Because its required liquidity is $F \cdot H(\frac{\theta^* - \theta}{\delta})$, we can obtain the bank failure threshold θ^T. θ^T might be lower than $\underline{\theta}$,[17] in which case we have the corner solution $\widehat{\theta}^i = \underline{\theta}$.

Cash: c	Calling creditors: $F \cdot H(\dfrac{\theta^* - \theta}{\delta})$
Illiquid asset: $\pi(\theta)\diagdown \begin{matrix}(1-c)X \\ 0\end{matrix}$	Staying creditors: $\pi(\theta)\diagdown \begin{matrix} R \cdot F \cdot \left[1 - H\left(\dfrac{\theta^* - \theta}{\delta}\right)\right] \\ 0 \end{matrix}$
	Equityholder: $\pi(\theta)\diagdown \begin{matrix}(1-c)X - R \cdot F \cdot \left[1 - H\left(\dfrac{\theta^* - \theta}{\delta}\right)\right] \\ 0\end{matrix}$

Figure 5: A bank's balance sheet position at T_1

It is worth emphasizing that a higher interbank rate leads to greater *downside risk* to bank creditors, but no upside risk. Concretely, in (5), a higher I leads to a higher $\widehat{\theta}^i$, meaning a greater probability of the bank's failure. However, conditional on the bank's survival $(\theta > \widehat{\theta}^i)$, a staying creditor does not benefit from a higher I, i.e., he still obtains the notional value R with probability $\pi(\theta)$ in (4); in fact, I does not appear inside the integral of (4). This result originates in that debt is a "fixed-income" security with an up limit in payoffs and that the asset payoff realizations at T_2 across banks are correlated (see Appendix on Setup in Section 2.1 for an illustration). That is, creditors in a bank suffer if their bank cannot borrow from the interbank market but cannot gain if their bank earns profits in interbank lending (i.e., all the profits accrue to the bank equityholder). This asymmetry in payoffs coupled with imperfect information about the type of a bank leads to creditors' reacting to a higher interbank rate by running more often.

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[17] Rigorously, this means that even for $\theta = \underline{\theta}$, strict inequality in (5a) holds, so no θ can make (5a) have equality.

We can solve the equation system to obtain the solution pair $(\theta^*, \widehat{\theta}^i)$ for given c, R and I. In conducting the comparative statics, we have Lemma 2.

Lemma 2 *With the interbank market, the creditor run equilibrium for an individual bank is characterized by the pair $(\theta^*, \widehat{\theta}^i)$, which solves the system of equations (4)-(5). The creditor-run game has a unique (stable) equilibrium under some parametric values. An increase in I leads to θ^* increasing along an upward spiral in that $\frac{\partial \widehat{\theta}^i}{\partial I} \geq 0$ (in (5)), $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ (in (4)), and $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} \geq 0$ (in (5)), with strict inequalities holding for non-corner solutions in (5).*

Proof: see Appendix.

The intuition behind the comparative statics in Lemma 2 is as follows. As in Lemma 1, the liquidity status of a bank impacts the *coordination risk* among its creditors in debt rollover decisions. *Ceteris paribus,* a rise in the funding cost in the interbank market means a deterioration of the liquidity status of a bank. A small rise in the interbank rate can significantly increase the coordination risk, and thus creditors rush for the exit. Specifically, an increase in the funding cost in the interbank market makes the bank's failure more likely ($\frac{\partial \widehat{\theta}^i}{\partial I} > 0$ in (5)), which in turn triggers a more conservative behavior of creditors ($\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ in (4)), which causes the bank's failure to be even more likely ($\frac{\partial \widehat{\theta}^i}{\partial \theta^*} > 0$ in (5)), and so on. Therefore, a small increase in I can result in a big rise in θ^*.

From Lemma 2, whether a bank actually fails depends on its fundamental value (its realized θ) and the interbank rate (I). This is consistent with the evidence in Gorton and Metrick (2011b) and Covitz et al. (2012).

The interbank market equilibrium   Because there are a continuum of banks in the system and their asset quality is i.i.d., the marginal bank in the system that survives at T_1 is

$$\widehat{\theta} = \widehat{\theta}^i \tag{6}$$

So, banks indexed $\theta \geq \widehat{\theta}$ survive and the others fail. Hence, the aggregate liquidity that survival banks possess is $\int_{\widehat{\theta}}^{\overline{\theta}} c\, d\theta$. For a bank with asset quality θ to survive, its required liquidity is

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$c^T(\theta) = F \cdot H(\frac{\theta^* - \theta}{\delta})$. So the market clearing commands that the following condition be true:

$$
\begin{cases}
\int_{\widehat{\theta}}^{\overline{\theta}} cg(\theta)d\theta = \int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^* - \theta}{\delta})g(\theta)d\theta & \text{if } \widehat{\theta} > \underline{\theta} \\
\int_{\widehat{\theta}}^{\overline{\theta}} cg(\theta)d\theta \geq \int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^* - \theta}{\delta})g(\theta)d\theta & \text{if } \widehat{\theta} = \underline{\theta}
\end{cases}
\tag{7}
$$

In (7), if $\widehat{\theta}$ is not the corner solution (i.e., $\widehat{\theta} > \underline{\theta}$), the aggregate amount of possessed liquidity must be equal to the aggregate amount of required liquidity. If $\widehat{\theta}$ is the corner solution (i.e., $\widehat{\theta} = \underline{\theta}$), the aggregate possessed liquidity can be excess.

To complete the above analysis, we need to show that banks indexed $\theta < \widehat{\theta}$ do not participate in the interbank market, i.e., neither borrow nor lend. Clearly, they don't borrow as they cannot afford.[18] We also assume that a bank that goes to bankruptcy at T_1 has no ability at T_2 to recover its investment if it lends in the interbank market, so it does not lend in the first place.[19]

From the above analysis, banks *endogenously* have three segments at T_1 according to their realized asset quality: lending banks with θ such that $F \cdot H(\frac{\theta^* - \theta}{\delta}) \leq c$, borrowing banks with θ such that $F \cdot H(\frac{\theta^* - \theta}{\delta}) > c$ and $\theta \geq \widehat{\theta}$, and failure banks with $\theta < \widehat{\theta}$.

We summarize the equilibrium at T_1 by Proposition 1.

Proposition 1 *With an interbank market, the equilibrium at T_1 is characterized by the triplet $(\theta^*, \widehat{\theta}, I)$, which solves the system of equations (4)-(5), (6) and (7) for given c and R.*

Proof: see Appendix.

In Proposition 1, there may exist more than one interbank market rate I that can clear the market for some c. In fact, when c is not very high, the marginal bank, $\widehat{\theta}$, satisfies $\widehat{\theta} > \underline{\theta}$. The interbank rate is uniquely determined by marginal bank $\widehat{\theta}$. The equilibrium rate in this case is equal to the marginal bank's equity value divided by its funding shortage. When c is high enough, the marginal bank is bank $\underline{\theta}$, in which case the equilibrium interbank rate can be any rate between

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[18] The borrowed liquidity and its own liquidity of such a bank is not enough to cover the demand of its creditors. The bank run will occur and its long-term asset will be destroyed, so no lenders are willing to lend liquidity to it in the first place.

[19] When a decision makes no difference to its equity value, a bank maximizes the value of its debtholders.

1 and the rate that marginal bank $\underline{\theta}$ can afford. When c is further higher, the interbank rate must be 1.

Proposition 1 gives the first main result of the paper – a new rationale for the interbank market. Earlier work such as Bhattacharya and Gale (1987) and Allen and Gale (2000) assume bank-specific liquidity shocks ex post and thus take illiquidity risk of banks as exogenous. In our model, the illiquidity risk of banks is *endogenous*, originating in bank-specific solvency shocks. Using our framework of the interbank market, we show that the rollover decision of creditors in an individual bank and the interbank rate are jointly determined in equilibrium, which in turn is dependent on the status of the aggregate economy.

4 Equilibrium at T_0

In this section, we study the equilibrium at T_0; in doing so we will endogenize c and R. We then conduct the efficiency and welfare analyses. We consider both the constrained second best equilibrium and the competitive equilibrium. Since banks are identical at T_0, to reduce notational clutter, we drop the bank index superscript i in this section, unless otherwise specified.

4.1 Constrained second best equilibrium

We state the equilibrium concept.

Constrained second best equilibrium A constrained second best equilibrium consists of the following three elements:

(i) For given (c, R) set at T_0, Proposition 1 determines equilibrium outcome $(\theta^*, \widehat{\theta}, I)$ at T_1.

(ii) A creditor demands an interest rate R at T_0 such that given (c, R) and thereby subsequent equilibrium outcome $(\theta^*, \widehat{\theta}, I)$, the creditor breaks even ex ante at T_0.

(iii) Knowing the response of $(R, \theta^*, \widehat{\theta}, I)$ to c, the social planner chooses an optimal c at T_0 to maximize the aggregate expected value of all banks in the economy.

Step 1: How R is determined

We first study element (ii). We can work out the *ex post* payoff to creditors and the equityholder (of the equilibrium outcome) in the creditor-run game. The ex post payoff to the equityholder in a

bank of quality θ at T_1 is calculated as follows. If the bank fails (i.e., $\theta < \widehat{\theta}$) at T_1, its equityholder gets nothing. If the bank survives (i.e., $\theta \geq \widehat{\theta}$) at T_1, the equityholder's payoff includes his claim from the long-term asset as well as his position in the interbank market, which is

$$\underbrace{\left[c - F \cdot H(\frac{\theta^* - \theta}{\delta}) \right] I}_{\text{Gain or loss in interbank market}} + \underbrace{\left[(1 - c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \right] \pi(\theta)}_{\text{Payoff in risky investment}} \qquad (8)$$

(8) is clear based on Figure 5. The first term, which can be positive (i.e., for lending) or negative (i.e., for borrowing), is the bank's gain or loss in the interbank market, and the second term is the expected payoff of its long-term asset net of the claim of its staying creditors. In fact, for the marginal bank with $\theta = \widehat{\theta}$, the equity value in (8) is exactly equal to 0, which can be verified by (5)-(5a). A bank with quality $\theta > \widehat{\theta}$ has a positive expected equity value.

Similarly, we can work out the *ex post* payoff to a creditor in the creditor-run game. If a bank with fundamental θ fails at T_1, the bank's total asset is the cash c, which is divided by the creditors who call. Hence, a creditor who calls obtains $\frac{c}{\Phi(\theta;\theta^*)} = \frac{c}{F \cdot H(\frac{\theta^* - \theta}{\delta})}$ by recalling that the proportion of creditors calling is $H(\frac{\theta^* - \theta}{\delta})$, and a creditor who does not call obtains 0. If a bank survives to T_2, a number of $F \cdot H(\frac{\theta^* - \theta}{\delta})$ creditors has called at T_1, each of whom obtains face value 1, while each staying creditor's expected payoff is $\pi(\theta)R$ at T_2.[20]

Thus, we can obtain the expected payoff to a creditor and thereby his ex ante participation condition at T_0. That is,

$$R_0 = \int_{\underline{\theta}}^{\widehat{\theta}} \left[\underbrace{\int_{-\infty}^{\theta^*} \frac{c}{F \cdot H(\frac{\theta^* - \theta^i}{\delta})} \cdot h(\theta^{ij}|\theta^i)d\theta^{ij}}_{\text{Call on failure bank}} + \underbrace{\int_{\theta^*}^{+\infty} 0 \cdot h(\theta^{ij}|\theta^i)d\theta^{ij}}_{\text{Hold on failure bank}} \right] g(\theta^i)d\theta^i$$

$$+ \int_{\widehat{\theta}}^{\overline{\theta}} \left[\underbrace{\int_{-\infty}^{\theta^*} 1 \cdot h(\theta^{ij}|\theta^i)d\theta^{ij}}_{\text{Call on survival bank}} + \underbrace{\int_{\theta^*}^{+\infty} \left(R \cdot \pi(\theta^i) \right) \cdot h(\theta^{ij}|\theta^i)d\theta^{ij}}_{\text{Hold on survival bank}} \right] g(\theta^i)d\theta^i. \qquad (9)$$

(9) gives the position for creditor j in bank i. Ex ante, at T_0, this creditor faces two levels of uncertainty: the quality of his bank and the signal that he will receive. The density $g(\theta^i)$ is

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[20] Staying creditors make their claim on the long-term risky asset of $1 - c$ units. We will show that in equilibrium $(1 - c)X > FR$. So a staying creditor obtains R if and only if the long-term asset realizes its high state payoff X.

about the first level of uncertainty and the conditional density $h(\theta^{ij}|\theta^i)$ is about the second level of uncertainty. The payoff in each scenario has been explained in the last paragraph.

Two remarks regarding the participation condition are in order. First, the four terms in (9) correspond to the four elements in Table 1, respectively. The difference in payoffs between Table 1 and (9) is minor when $w_0 = 1$ and $\gamma = 1$. That is, without assuming the involvement of a fund manager, the simplified payoff structure of bank runs under $w_0 = 1$ and $\gamma = 1$ in Table 1 is a close approximation of the full payoff structure in (9). From (9), we see that calling loans at T_1 makes a creditor either fully recover the face value of investment 1 (in case of bank survival) or suffer a small loss (in case of bank failure). As in Rochet and Vives (2004) and Morris and Shin (2009), we have assumed that a fund manager obtains a constant payoff, w_0, in these two cases.

Second, the participation condition (9) is in terms of the *gross* payoff to a creditor. The intuition is that when a creditor decides on his participation R, he takes account of that his fund manager's objective is not entirely aligned with his. In particular, under the assumption that $w_0 \to 0$, $\gamma \to 0$ and $\frac{w_0}{\gamma} = 1$, the net payoff to a creditor (after the deduction of the compensation to his fund manager) approaches his gross payoff.[21]

Equation (9) can be simplified and rewritten as

$$R_0 = \int_{\underline{\theta}}^{\widehat{\theta}} (\frac{c}{F}) \cdot g(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[H(\frac{\theta^* - \theta}{\delta}) \cdot 1 + \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \cdot (R \cdot \pi(\theta)) \right] g(\theta)d\theta. \qquad (9')$$

(9') is very intuitive. Conditional on a bank failing, the expected payoff to a creditor is $\frac{c}{F}$. This is because that c will be distributed among the calling creditors while ex ante all creditors have an equal chance to end up as calling creditors; therefore, ex ante, this is equivalent to c being distributed equally among a number of F creditors. Conditional on the bank surviving, with probability $H(\frac{\theta^* - \theta}{\delta})$, a creditor receives a bad signal and thus calls, in which case his payoff is 1; with probability $1 - H(\frac{\theta^* - \theta}{\delta})$, he receives a good signal and thus stays, in which case his expected payoff is $R \cdot \pi(\theta)$.

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[21] We can write the participation condition in terms of the *net* payoff to a creditor. That is, a creditor's payoff is the payoff from his investment net of the compensation to his fund manager. In this case, the payoffs for the four scenarios in (9) are replaced by $\frac{c}{F \cdot H(\frac{\theta^* - \theta^i}{\delta})} - w_0$, 0, $1 - w_0$, and $(1 - \gamma)(R \cdot \pi(\theta^i))$, respectively. This alternative way of the participation condition does not change the model results qualitatively.

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For cleanness, we might explicitly add another constraint:

$$FR < (1-c)X. \tag{10}$$

Constraint (10) gives a sufficient condition to guarantee that creditors are repaid with R (no default) if and only if the long-term asset realizes its high state cash flow X at T_2. This condition, however, is not necessary in general when the optimization problem, to be shown shortly, is taken into account. That is, it is not optimal in equilibrium to choose too high a c.

Lemma 3 summarizes the equilibrium for a given c.

Lemma 3 *For a given c, the equilibrium is characterized by the vector $(R, \widehat{\theta}, \theta^*, I)$, which solves the system of equations (4)-(5), (6), (7) and (9'), and satisfies (10).*

Proof: see Appendix.

Step 2: How c is determined

Now we study the decision of the social planner at T_0, element (iii) of the constrained second best equilibrium. The social planner's objective is to maximize the total social surplus. The social planner's problem is

$$\max_{c} \quad \int_{\underline{\theta}}^{\widehat{\theta}} cg(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[c + (1-c)(X \cdot \pi(\theta))\right] g(\theta)d\theta \qquad \text{(Program 1)}$$
$$\text{s.t.} \quad \text{(4)-(5), (6), (7), (9') and (10)}$$

In Program 1, the objective function is to maximize the aggregate value of all banks in the economy, including the failure banks at T_1 (the first term) and survival banks at T_2 (the second term). This aggregate value, in the end, is divided between the equityholders and the creditors in the economy. In fact, the aggregate value in the objective function is exactly equal to the sum of the equityholder's payoff across banks in (8) plus the sum of debtholders' payoffs across banks on RHS of (9') (see the proof of Lemma 4 in Appendix).[22]

In solving Program 1, we can think that there are two steps. First, for a given c, we obtain each of R, θ^* and $\widehat{\theta}$ as a function of c by Lemma 3. In particular, we obtain function $\widehat{\theta}(c)$. Second, by

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[22]Program 1 is equivalent to maximizing the aggregate equity value. The equivalence is because creditors in a bank, in total, claim a *constant residual* value, FR_0.

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plugging function $\widehat{\theta}(c)$ into the objective function of Program 1, we solve for the optimal c. Lemma 4 summarizes the result.

Lemma 4 *The constrained second best equilibrium is characterized by the vector* $(c, R, \widehat{\theta}, \theta^*, I)$, *which solves Program 1.*

Proof: see Appendix.

We explain the intuition behind the optimal c in Lemma 4. A higher c results in more banks surviving at T_1 (i.e., a lower $\widehat{\theta}$), but also less investment in long-term projects in the economy. The tradeoff leads to an optimal liquidity ratio at T_0. Specifically, denoting the aggregate bank value by V in the objective function in Program 1, the first order derivative of V is

$$\frac{\partial V}{\partial c} = \underbrace{\left\{ (-\frac{\partial \widehat{\theta}}{\partial c}) \cdot \left[(1-c)(X \cdot \pi(\widehat{\theta}))g(\widehat{\theta}) \right] \right\}}_{\text{more banks survive ex post}} - \underbrace{\left[\int_{\widehat{\theta}}^{\overline{\theta}} (X \cdot \pi(\theta))g(\theta)d\theta - 1 \right]}_{\text{higher return of banks ex ante}}.$$

The deadweight loss when a bank of quality θ fails at T_1 is $(1-c)(X \cdot \pi(\theta))$, so the gain from making more banks survive in the economy by holding one more unit of cash at T_0 is $(-\frac{\partial \widehat{\theta}}{\partial c}) \cdot \left[(1-c)(X \cdot \pi(\widehat{\theta}))g(\widehat{\theta}) \right]$. On the other hand, storing cash for banks means that valuable investment opportunities are wasted in the economy, the loss being $\int_{\widehat{\theta}}^{\overline{\theta}} (X \cdot \pi(\theta))g(\theta)d\theta - 1$. The tradeoff leads to an optimal level of cash holdings for banks at T_0.

4.2 Competitive equilibrium (*laissez-faire* equilibrium)

In the previous subsection, a bank's ex ante cash holdings are determined by the social planner. In this subsection, we study the competitive market equilibrium: banks decide their ex ante liquidity holdings based on their rational choice (individual rationality).

Lemma 3 shows that for given ex ante cash holdings c, the market force determines the equilibrium outcome $(R, \widehat{\theta}, \theta^*, I)$ subsequently. Now we ask: if an individual bank takes the prices $\{R, I\}$ and creditors' response $\{\theta^*\}$ given, how much liquidity it is willing to store in the first place.

Formally, we have the following equilibrium concept.

Competitive equilibrium A competitive equilibrium consists of the following two elements:

(i) Suppose *all* banks in the system choose cash holdings at T_0 as c. Lemma 3 determines equilibrium outcome $(R, \widehat{\theta}, \theta^*, I)$.

(ii) Taking market prices $\{R, I\}$ and creditors' strategy at T_1, θ^*, as given and fixed, an *individual* bank chooses its optimal cash holdings as being c.

We turn to element (ii). An individual bank's ex ante optimization problem in storing liquidity is

$$c = \operatorname*{argmax}_{c^i} \int_{\widehat{\theta}^i(c^i)}^{\bar{\theta}} \left\{ \left[c^i - F \cdot H(\frac{\theta^* - \theta}{\delta}) \right] I + \left[\begin{array}{c} (1 - c^i)X - \\ R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \end{array} \right] \pi(\theta) \right\} g(\theta) d\theta \tag{11}$$

where $\quad \widehat{\theta}^i(c^i) = \max(\underline{\theta}, \theta^T(c^i))$, and $\theta^T(c^i)$ solves

$$\frac{\left[(1 - c^i)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta^T}{\delta}) \right) \right] \pi(\theta^T)}{I} + c^i = F \cdot H(\frac{\theta^* - \theta^T}{\delta}) \tag{11a}$$

In the above, c^i denotes the cash holding choice for an *individual* bank. In the objective function of (11), a bank's aim is to maximize its expected equity value, where the equity value for a realization of θ is shown in (8). The key to understanding the optimization problem is the liquidity budget constraint, (11a). In fact, constraint (11a) is the same as (5)-(5a), with the only difference that now c^i is endogenously chosen by an individual bank. When the interbank market exists, a bank rationally anticipates that it can borrow funding in the interbank market, on top of its own liquidity. Importantly, an individual bank takes the interbank rate I as given. Thus, when a bank stores liquidity c^i, it anticipates that it survives at T_1 if and only if its realized θ satisfies $\theta \geq \widehat{\theta}^i(c^i)$, where $\widehat{\theta}^i(c^i)$ is defined in (11a).

It is worth noting that $\widehat{\theta}^i(c^i)$ and $\widehat{\theta}$ have different meanings. $\widehat{\theta}$ is the general equilibrium outcome, while $\widehat{\theta}^i(c^i)$ is the survival threshold in the partial equilibrium that an *individual* bank can influence by changing c^i while taking the interbank rate I as given.

The competitive equilibrium means that the liquidity holding decisions of banks need to be *incentive compatible*. That is, there is a fixed point problem: $c \to (R, \widehat{\theta}, \theta^*, I) \to c$. We have the following result.

Lemma 5 *The competitive equilibrium is characterized by the vector $(c, R, \widehat{\theta}, \theta^*, I)$, which solves the system of equations (4)-(5), (6), (7), (9') and (11), and satisfies (10). When $F \leq \underline{F}$, where*

$\underline{F}$ is a cutoff, the competitive equilibrium exists and is unique. When $F > \underline{F}$, the competitive equilibrium does not exist.

The competitive equilibrium has the following properties:

1) The level of cash holdings c is such that it exactly achieves the ex post efficiency (i.e., $\widehat{\theta} = \underline{\theta}$). That is, no banks are expected to fail at T_1.

2) The equilibrium interbank rate is $I = \int_{\underline{\theta}}^{\overline{\theta}} [X \cdot \pi(\theta)] g(\theta)d\theta$.

Proof: see Appendix.

Lemma 5 states that if the competitive equilibrium exists, it must correspond to a corner solution to the system of equations. We explain the intuition behind Lemma 5. Denote the equity value in the objective function of (11) by V_E. The first order derivative on V_E is

$$\frac{\partial V_E}{\partial c^i} = \int_{\widehat{\theta}^i(c^i)}^{\overline{\theta}} [I - X \cdot \pi(\theta)] g(\theta)d\theta.$$

If $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$ is not true, we can find that the optimum c^i is either 0 or 1. That is, an individual bank prefers to either hold all cash or invest all cash in its long-term asset, which is also the result in Diamond and Rajan (2005). The driving force behind these extreme decisions is that an individual bank takes I as given and being constant, with the outcome of the "constant returns to scale" effect. In this case, the competitive equilibrium does not exist because the fixed point problem of $c \rightarrow (R, \widehat{\theta}, \theta^*, I) \rightarrow c$ does not have a solution. However, the competitive equilibrium exists and is unique when the following two conditions are satisfied: i) $I = \int_{\underline{\theta}}^{\overline{\theta}} [X \cdot \pi(\theta)] g(\theta)d\theta$ and ii) $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$. The reason is the following. These two conditions together imply that $\frac{\partial V_E}{\partial c^i} \equiv 0$, that is, an individual bank can choose any c as its optimum cash holdings, including the one that can achieve the ex post efficiency (i.e., $\widehat{\theta} = \underline{\theta}$) as in Proposition 1. In turn, given this level of cash holdings, any rate between 1 and $I(\underline{\theta})$ (where $I(\underline{\theta})$ is the rate that bank $\underline{\theta}$ can afford) supports the equilibrium by Proposition 1, while the interbank rate given by conditions i) and ii) belongs to this region. Therefore, conditions i) and ii) guarantee the existence and the uniqueness of equilibrium. When F is not too high, the above two conditions are simultaneously satisfied.

4.3 Comparison between the second best equilibrium and the competitive equilibrium

In comparing the two equilibria at T_0, we have the following result:

Proposition 2 *The constrained second best, in general, cannot be implemented by the competitive equilibrium. There is an over-storage of liquidity (c) in the competitive equilibrium relative to the second best.*

Proof: see Appendix.

As shown in Lemma 5, the competitive equilibrium achieves the ex post efficiency. That is, the level of cash holding c is such that all banks can survive at T_1. However, the second best equilibrium trades off the ex ante efficiency (i.e., a higher expected return of long-term investment) and the ex post efficiency (i.e., a higher survival rate of banks). In general, the second best equilibrium commands that the ex post efficiency needs to be partially sacrificed. That is, the optimal cash holdings should be such that not all banks are saved at T_1. In other words, the cash holdings in the second best should be lower than in the competitive equilibrium.

Proposition 2 is somewhat counterintuitive. We have three remarks. 1) Proposition 2 might shed light on the phenomenon of the financial system being "awash with liquidity" prior to the recent crisis (see, e.g., Brunnermeier et al. (2009), Shin (2010)), which quickly turned into a "liquidity crunch" during the crisis (after an adverse shock hit). In our model, the *ex ante* being "awash with liquidity" can exacerbate the *ex post* "liquidity crunch" once a small adverse aggregate shock hits (through the channel of R; see next section). 2) The recent work of Hart and Zingales (2013) shows the similarly counterintuitive result that "consumers over-hoard liquidity", and their mechanism is similar to ours. 3) What we observe in reality is the outcome of the joint force of many factors whereas our theoretical model studies just one factor. For example, banks' under-storage of liquidity, if empirically supportive, can be simply because another important factor also exists - banks' expectation of government bailouts in a crisis (i.e., moral hazard issues).

We discuss the intuition for why the competitive equilibrium is not constrained efficient. With an interbank market, an individual bank trades off two investment opportunities ex ante at T_0: investing in its long-term asset or saving cash to make interbank lending later. We can show that in the constrained efficient equilibrium with $\widehat{\theta} > \underline{\theta}$, the interbank rate is higher than the return to the long-term investment. Intuitively, the social planner prefers banks to hold less cash ex ante in maximizing the aggregate value of all banks, with the consequence that some high-quality banks demand liquidity ex post and thus push up the interbank rate. Therefore, if there are no regulations,

the constrained second-best equilibrium cannot be supported (not incentive compatible): every bank tends to store more liquidity and to invest less in long-term assets, resulting in the over-storage of liquidity in the system in reaching competitive equilibrium.

What is behind the market failure? The market failure at T_0 – inefficient cash holdings – is due to the friction of uncontractibility of banks' status (idiosyncratic shocks) ex ante. In fact, under this friction, the ex ante contracting of liquidity sharing (co-insurance) among banks is impossible.

Concretely, because of the uncontractible idiosyncratic shocks among banks, there is a wealth redistribution (reallocation) among banks at T_1. A 'lucky' bank that realizes a higher fundamental and has sufficient cash can use its excess cash to earn high return in interbank lending. Every individual bank wants to be a lucky one, and holding more cash can increase the chance. In fact, at the extreme when all other banks hold little cash and hence compete for liquidity at T_1, an individual bank that holds more cash in this case can earn huge profits. However, from the perspective of the social planner, wealth redistribution among banks at T_1 is a zero-sum game. He may prefer banks to *collectively* hold less cash ex ante in maximizing the aggregate value and let a few 'lucky' banks with high asset quality control the most part of the value ex post. That is, the objective function of an individual bank is not entirely aligned with that of the social planner. On the contrary, if idiosyncratic shocks are contractible and can be insured among banks, the two objective functions would be aligned, as in Program 1. Put slightly differently, in the frictioned world, when an individual bank holds more cash this has *negative* externality to other banks because other banks can earn less in interbank lending.[23] This pecuniary externality is close in spirit to the analysis of Hart and Zingales (2013) and Acharya, Shin and Yorulmazer (2013).

In Bhattacharya and Gale (1987), depending on parameters, an over-storage of liquidity in the banking system is also possible as it is in our paper (see Propositions 8 and 9 in their paper).[24] In fact, the key friction driving the market failure (inefficient liquidity holdings) in the two papers is similar, namely, the ex ante contracting of liquidity sharing (co-insurance) among banks is impossible. Nevertheless, there are two key differences between the two papers, which determine that

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[23] This is both because fewer banks in the economy suffer liquidity shortage and because fewer long-term assets are in these banks' balance sheets.

[24] Also see Ahnert (2013) on the efficiency of liquidity holdings.

over-investment of liquidity is the dominating phenomenon in our paper. First, in their model, shocks are *preference shocks* and banks are distinguished by the proportion of "early consumers", so banks cannot be *ranked* (i.e., good banks or bad banks). In comparison, the liquidity shock in our paper is *endogenous*, originating in the (idiosyncratic) asset quality shock. A bank with higher quality not only has a higher fundamental value of asset but also fewer creditors to withdraw earlier. So banks in our model can be ranked ex post and in fact the "wealth inequality" among banks is high. Second, banks in our model are of a continuum of types and ex post there are three segments (lending banks, borrowing banks and failure banks), while in Bhattacharya and Gale (1987) banks are only of two types and ex post there are two segments (lending banks and borrowing banks). Both factors above lead to banks tending to hold more liquidity ex ante in our model – to reduce the chance to be "worst banks".

In the next section, we study inefficiency at T_1, originating in the friction of *information asymmetry* about banks' status at T_1. A small negative aggregate shock on asset quality in the system can trigger the joint event of "liquidity evaporation" and "interbank market freezing".

5   Aggregate shock: crisis, contagion, and amplification

In this section, we study our model under an aggregate shock, which might help in understanding the origin of crisis, contagion and amplification.

The creditor run and the interbank market run   Before proceeding to the analysis, we derive further results about the equilibrium at T_1. As this section is to study the amplification effect under a shock, we focus on studying the interior solution of the equilibrium at T_1. In fact, a shock on the state of nature at T_1 will lead to the interior solution of equilibrium even if the original equilibrium without the shock is the corner solution under the competitive equilibrium.

The equilibrium at T_1 contains two run equilibria: the creditor run equilibrium and the interbank market run equilibrium. We analyze them one by one.

The creditor run equilibrium for an individual bank is given by the system of equations (4)-(5). We have explained it in detail earlier. For ease of exposition, here we repeat it for the case with

the non-corner solution:

$$\int_{\underline{\theta}}^{\widehat{\theta}^i} 0 \cdot h_g(\theta|\theta^*)d\theta + \int_{\widehat{\theta}^i}^{\overline{\theta}} R \cdot \pi(\theta) \cdot h_g(\theta|\theta^*)d\theta = \frac{w_0}{\gamma} \tag{12a}$$

$$\frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^*-\widehat{\theta}^i}{\delta})\right)\right]\pi(\widehat{\theta}^i)}{I} + c = F \cdot H(\frac{\theta^*-\widehat{\theta}^i}{\delta}) \tag{12b}$$

(12a) and (12b) correspond to (4) and (5), respectively. (12a)-(12b) results in a unique θ^* for a given I. Also, an increase in I leads to θ^* rising along an upward spiral in that $\frac{\partial \widehat{\theta}^i}{\partial I} > 0$ (in (12b)), $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ (in (12a)), and $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} > 0$ (in (12b)).

We focus on explaining the "interbank market run". The "interbank market run" equilibrium is given by the system of equations (13a)-(13b):

$$\int_{\widehat{\theta}}^{\overline{\theta}} cg(\theta;\mu)d\theta = \int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^*-\theta}{\delta})g(\theta;\mu)d\theta \tag{13a}$$

$$I = \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^*-\widehat{\theta}}{\delta})\right)\right]\pi(\widehat{\theta})}{F \cdot H(\frac{\theta^*-\widehat{\theta}}{\delta}) - c} \tag{13b}$$

(13a) is from (7), and (13b) is from (6) and (5). (13a)-(13b) results in a unique I for a given θ^*. Intuitively, each bank responds to creditors' strategy θ^* and chooses its own liquidity hoarding $F \cdot H(\frac{\theta^*-\theta}{\delta})$. The demand and supply forces in the interbank market determine the unique interbank rate I. When θ^* is high, the interbank market suffers a "run" by banks because banks are less willing to lend.

Concretely, we can think that the interbank market run determines the interbank rate in two steps. First, the demand and supply forces determine the marginal bank, that is, for a given θ^*, $\widehat{\theta}$ is uniquely determined in (13a). Second, the marginal bank determines the equilibrium interbank rate, that is, for given $\widehat{\theta}$ (and θ^*), I is uniquely determined in (13b). We explain the intuition. Let us consider the position of the marginal bank $\widehat{\theta}$: if it does not borrow from the interbank market, it fails and consequently its equityholder gets nothing; if it borrows an amount $c^T(\widehat{\theta}) - c = F \cdot H(\frac{\theta^*-\widehat{\theta}}{\delta}) - c$, the difference between its required liquidity and its own liquidity, the expected payoff of its equityholder is $\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^*-\widehat{\theta}}{\delta})\right)\right]\pi(\widehat{\theta})$. Hence, the maximum *risk-adjusted* interbank rate that the marginal bank is willing to and is able to pay is $\frac{\left[(1-c)X-R\cdot F\cdot\left(1-H(\frac{\theta^*-\widehat{\theta}}{\delta})\right)\right]\pi(\widehat{\theta})}{F\cdot H(\frac{\theta^*-\widehat{\theta}}{\delta})-c}$, denoted by $I(\widehat{\theta})$. Therefore, the equilibrium interbank rate must be

$I(\widehat{\theta})$, due to the nature of the *competitive* interbank market. In fact, if the interbank rate is higher than this level, this marginal bank as well as the banks of lower quality cannot afford the liquidity, so the total supply of liquidity would exceed the total demand. On the other hand, if the interbank rate is lower, some banks of lower quality can afford and thus demand and compete for liquidity, so the total demand would exceed the total supply. In short, the equilibrium rate must be equal to the marginal bank's equity value divided by its funding shortage. It is important to emphasize, nevertheless, that in terms of economic mechanism, $(\widehat{\theta}, I)$ are *simultaneously* determined. As in any general equilibrium model, prices and the allocation must be jointly determined in equilibrium.

We have Lemma 6 for the interbank market run equilibrium.

Lemma 6 *The interbank market run equilibrium is characterized by (13a)-(13b), which gives a unique interbank rate I for a given θ^*. Under a sufficient condition that c is high enough, we have the comparative statics that $\frac{\partial I}{\partial \theta^*} > 0$ and $\frac{\partial I}{\partial \mu} < 0$.*

Proof: see Appendix.

The intuition behind the comparative statics in Lemma 6 is easy to understand. The more conservative the creditors are (i.e., a higher θ^*), the more liquidity banks hoard (i.e., a higher $F \cdot H(\frac{\theta^* - \theta}{\delta})$ for every given θ). Banks run on the interbank market more strongly, and hence the equilibrium interbank rate I is higher. When there is an aggregate negative shock on asset quality (i.e., a lower μ), some banks need more liquidity than they expect. This means either that the aggregate demand for liquidity in the interbank market increases or that the aggregate supply decreases, which pushes up the interbank rate.

To summarize, we have:

Lemma 7 *The creditor run equilibrium ((12a)-(12b)) results in a unique θ^* for a given I, while the interbank market run equilibrium ((13a)-(13b) determines a unique I for a given θ^*. The equilibrium in the system at T_1 is characterized by the fixed point problem between θ^* and I.*

Crisis and amplification So far we have assumed that there is no aggregate uncertainty but only idiosyncratic shocks to banks, that is, the distribution $g(\cdot)$ is given and deterministic.

Now we consider that there is aggregate uncertainty. Specifically, as in Allen and Gale (2000) and Diamond and Rajan (2005), we allow a zero-probability event to occur. There are two states of nature at T_1 for the aggregate economy: normal state ($s = N$) and extreme state ($s = \varepsilon$). Ex ante, at T_0, the extreme state will occur with negligible probability (in the limit, probability zero) and the normal state will occur *almost surely*. If the normal state occurs, the asset quality distribution is given by $g_N(\cdot)$ while in the extreme state the distribution is $g_\varepsilon(\cdot)$, where g_N has first-order stochastic dominance over g_ε (that is, their c.d.f. satisfies that $G_N(\theta) < G_\varepsilon(\theta)$ for any $\theta \in (-\infty, +\infty)$). Following the previous example, we can assume that in state N the distribution of θ is $\theta \sim N(\mu, \sigma^2)$ and in state ε it is $\theta \sim N(\mu_\varepsilon, \sigma^2)$ where $\mu_\varepsilon < \mu$; that is, $g_N(\theta) = \phi(\theta; \mu, \sigma^2)$ and $g_\varepsilon(\theta) = \phi(\theta; \mu_\varepsilon, \sigma^2)$.

The aggregate uncertainty can be interpreted in an alternative way. As in the business cycle literature (e.g., Kiyotaki and Moore (1997)), the occurrence of state ε is equivalent to the economy suffering an *unexpected* aggregate shock (away from the normal state N).

In Lemmas 4 and 5, we have shown that for a given distribution $g(\cdot)$, we have the equilibrium outcome $(c, R, \widehat{\theta}, \theta^*, I)$. So, if agents can perfectly foresee the aggregate state, N or ε, we have the corresponding equilibrium, denoted by $(c_s, R_s, \widehat{\theta}_s, \theta^*_s, I_s)$, where $s = N$ and ε.

At T_0, agents are uncertain about the aggregate state at T_1. However, because the extreme state will occur with negligible probability, it has no effect on the allocation at T_0. The allocation is completely determined by the distribution $g_N(\cdot)$. That is, the allocation at T_0 is the vector (c_N, R_N), the cash holdings being c_N and the interest rate being R_N in the deposit contract.

Now we examine what will happen under this allocation when the small-probability event (i.e., the extreme state) occurs at T_1. Under the shock, the equilibrium at T_1 is given by the system of equations (12a)-(12b) and (13a)-(13b), where the allocation (c, R) is (c_N, R_N) while the asset quality distribution $g(\cdot)$ is replaced by $g_\varepsilon(\cdot)$ in (13a) and the public information in (12a) is also replaced by $g_\varepsilon(\cdot)$ (so the posterior (conditional) density of θ in (12a) is $h_{g_\epsilon}(\theta|\cdot)$). The new equilibrium outcome $(\widehat{\theta}, \theta^*, I)$ is denoted by $(\widehat{\theta}_{N\varepsilon}, \theta^*_{N\varepsilon}, I_{N\varepsilon})$.
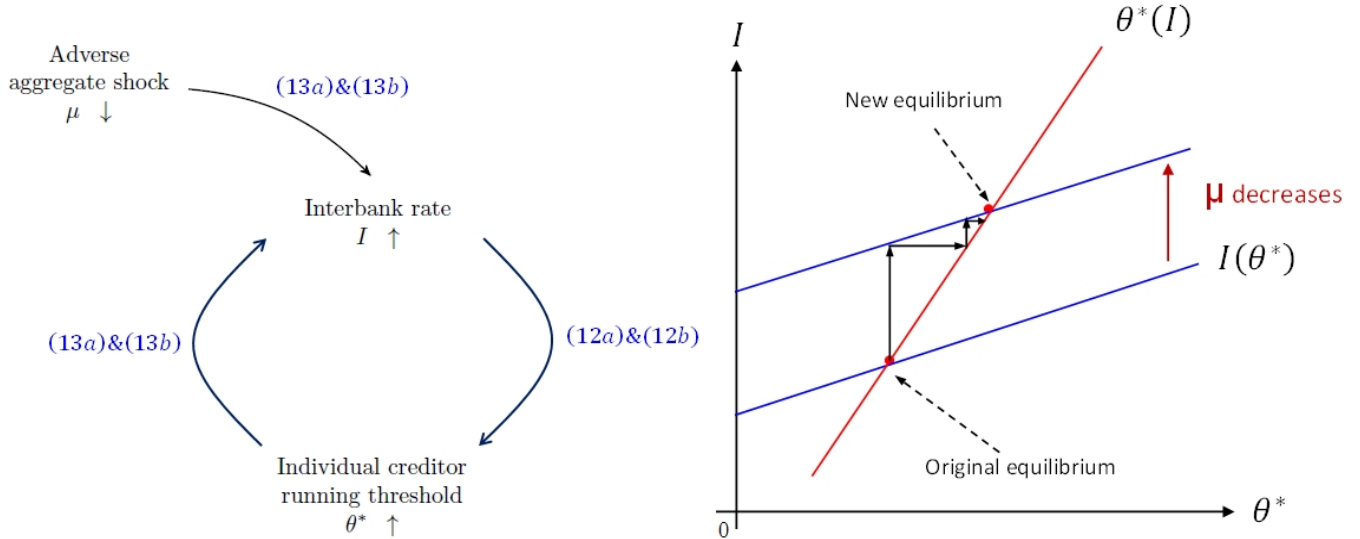
We have the following result.

Proposition 3 *Under the allocation* $(c, R) = (c_N, R_N)$ *for the normal state, when the shock (extreme state) occurs at T_1 (i.e., $g = g_\varepsilon$), the equilibrium, characterized by $(\widehat{\theta}_{N\varepsilon}, \theta^*_{N\varepsilon}, I_{N\varepsilon})$, is determined by the system of equations (12a)-(12b) and (13a)-(13b). We have that $\widehat{\theta}_{N\varepsilon} > \widehat{\theta}_N$, $\theta^*_{N\varepsilon} > \theta^*_N$ and $I_{N\varepsilon} > I_N$. The aggregate shock triggers a reinforcing spiral between creditor runs and the interbank market run in reaching the new equilibrium in that $\frac{\partial I}{\partial \mu} < 0$ in (13a)-(13b), $\frac{\partial \theta^*}{\partial I} > 0$ in (12a)-(12b), and $\frac{dI}{d\theta^*} > 0$ in (13a)-(13b).*

Proof: see Appendix.

It is easy to show that under the 'surprising' shock, the bank failure threshold, the calling threshold of creditors and the interbank rate all increase and are higher than expected, i.e., $\widehat{\theta}_{N\varepsilon} > \widehat{\theta}_N$, $\theta^*_{N\varepsilon} > \theta^*_N$ and $I_{N\varepsilon} > I_N$.

Our focus is to understand the spiraling effect triggered by the shock. The adverse shock $g_N \to g_\varepsilon$ (or $\mu \downarrow \mu_\varepsilon$) has two channels of impact. The first is the *individual (local) effect*, which corresponds to the public information changing from g_N to g_ε in (12a). Intuitively, for any individual creditor in any bank, he receives bad news about the asset quality of his own bank. In reality, the ABX index might be such public information. The second is the *systemic effect*, which corresponds to the asset quality distribution in the system changing from g_N to g_ε in (13a). The adverse shock triggers a chain of actions and reactions in the system and impacts the asset price - the interbank rate. Specifically, there is a feedback spiral between the interbank market run and creditor runs, as illustrated by Figure 6.

Figure 6: Feedback loop and spiral

Basically, there is a feedback loop between I and θ^*. A higher interbank rate exacerbates creditor runs. In turn, a higher chance of creditor run leads to *all* banks hoarding more liquidity, increasing the interbank rate.[25] Empirically, I measures interbank market freezing and θ^* measures liquidity evaporation in the system. Therefore, interbank market freezing and liquidity evaporating reinforce each other, which explains the unprecedented event in the recent crisis discussed at the beginning of this paper. Put slightly differently, an adverse aggregate shock triggers the reinforcing spiral between the decrease in *supply* of liquidity (i.e., the interbank run) and the increase in *demand* for liquidity (i.e., creditor runs) in the system.[26]

It is not surprising that an unanticipated negative event (shock) can have adverse consequences, as the name suggests. What we emphasize here, however, is the *amplification and spiraling* mechanism, that is, why a small shock can have a large impact.

We have compared the equilibrium outcome $(\widehat{\theta}_{N\varepsilon}, \theta^*_{N\varepsilon}, I_{N\varepsilon})$ with $(\widehat{\theta}_N, \theta^*_N, I_N)$. In another way, it is helpful to compare the equilibrium outcome $(\widehat{\theta}_{N\varepsilon}, \theta^*_{N\varepsilon}, I_{N\varepsilon})$ with $(\widehat{\theta}_\varepsilon, \theta^*_\varepsilon, I_\varepsilon)$. That is, if agents

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[25] It is important to emphasize that our model of the interbank market is a Walrasian economy, not a game; the actual agents (banks) are infinitesimal price-takers, not strategic players. The price - the interest rate I - plays a key role in determining the equilibrium, as shown in Figure 6.

[26] Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) offer the insight of the spiral between market liquidity and funding liquidity. Our result differs from theirs in that the interbank rate impacts *coordination risk* for creditors in institutions rather than the leverage or margin constraints.

could foresee the adverse aggregate shock, how much better would the equilibrium outcome be? In anticipation of the extreme state, the equilibrium allocation at T_0 should be $(c_\varepsilon, R_\varepsilon)$. Comparing this allocation with the allocation without preparation for the extreme state, we can show that both the cash holdings and the interest rate are too low, i.e., $c_N < c_\varepsilon$ and $R_N < R_\varepsilon$. Therefore, we can imagine the following scenario: at T_0 agents foresee the state ε and set the allocation $(c_\varepsilon, R_\varepsilon)$, and at T_1 there is no shock on the state but instead a shock on the allocation: $(c_\varepsilon, R_\varepsilon) \downarrow (c_N, R_N)$, which means that some cash reserves are wiped out and the deposit rate is cut. We ask: what the consequences are under this shock. Clearly, an insufficient cash reserve in the system at T_1 can trigger a big crisis. Also, *ceteris paribus,* a lower interest rate means a lower stake of a creditor in his bank and thus a more severe coordination problem between creditors, and a creditor more likely runs for the exit (see Lemma 1). Therefore, there are two joint forces (both the too-low cash holdings and the too-low interest rate) leading to higher $\widehat{\theta}$, θ^* and I. This analysis may shed light on why debt runs and the liquidity crisis itself in 2007-2009 were so severe. It could have been in part because the pre-crisis interest rate was too low in the contract between financial institutions (e.g., shadow banks) and their investors. In particular, because R is in the *denominator* in determining θ^* (see (3)), a slight decrease in R can raise θ^* significantly. The ex ante being "awash with liquidity", leading to a lower equilibrium interest rate, can exacerbate the ex post "liquidity crunch" once an unexpected adverse shock hits.

Contagion　Our model also has implications on contagion, which comes through the channel of financial institutions "fishing from the same pool of short-term funding". In our model, a lower-quality bank fails not because it suffers a loss in its asset and thus its capital when investing in other banks, or because its own asset suffers a negative shock and thus it has to increase the demand of liquidity. Rather, the key problem can be that some higher-quality banks in the system suffer a negative shock on their asset and thus need more liquidity than expected (in order to reduce their illiquidity risk). The decrease in the net supply of liquidity of these higher-quality banks leads to a shrinking of liquidity in the pool, which actually hurts lower-quality banks first and most. In other words, the negative shock can be on higher-quality banks but lower-quality banks suffer first. In fact, in the financial crisis of 2007-2009, the crisis originated in the US, where there had been a subprime mortgage crisis. However, the first bank that suffered bank runs was Northern Rock, a UK bank.

6 Related literature

This paper provides a framework for studying the interbank market. We show the interdependence of the interbank market run and creditor runs.[27] The paper is related to several strands of the literature.

Our paper is related to the papers that use global game methods to address illiquidity risk (e.g., Morris and Shin (2004a, 2009), Rochet and Vives (2004), Goldstein and Pauzner (2005), Liu and Mello (2011), Eisenbach (2011)).[28] Rochet and Vives (2004) and Goldstein and Pauzner (2005) are the first to apply global games to study bank runs. Vives (2013) builds a general model to incorporate the results in this literature. Our paper contributes to this literature in that we study the interplay between illiquidity risk and insolvency risk in a *financial system* context and analyze systemic effects that played a large role in the recent crisis. We explicitly model the interbank market, and examine the transmission of shocks across institutions through the interbank market.[29] Notably, Morris and Shin (2009) build an analytical framework to separate the creditor risk in a financial institution into insolvency risk and illiquidity risk. We provide a general equilibrium treatment on Morris and Shin (2009) (as well as Liu and Mello (2011)) and endogenize some key parameters in their model.

Our paper is related to the contributions that study interbank markets (e.g., Bhattacharya and Gale (1987), Rochet and Tirole (1996), Allen and Gale (2004b), Freixas and Holthausen (2005)), and contagion through interbank claims (e.g., Allen and Gale (2000), Freixas et al. (2000), Dasgupta

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[27]Brunnermeier (2009) discusses various amplification mechanisms in the recent crisis. Using a game-theoretic approach, Liu and Mello (2012) study the amplification mechanism in the interbank market by emphasizing the role of a large lender; our model has a different mechanism. Uhlig (2010) examines a systemic bank run but does not study the role played by the interbank market and the feedback.

[28]In a related model not using global games, He and Xiong (2012) study the inter-temporal coordination problem among creditors.

[29]Eisenbach (2011) constructs a model to endogenize the liquidation value (or the "fire-sale penalty" in the language of Vives (2013)) by assuming an exogenous demand curve of the asset. Our work differs from his in at least two important ways: First, we study and endogenize the *competitive* interbank market. Second, the ex ante problem of banks is studied, so the aggregate liquidity in the system is *endogenously* determined and the efficiency (welfare) analysis is allowed. Liu and Mello (2011) study the optimal cash holdings in the presence of coordination risk for a single institution, which is a partial equilibrium framework. Morris and Shin (2008) discuss financial regulation in a system context.

(2004)). This literature typically studies the shocks on the liability side, e.g., Bhattacharya and Gale (1987) study interbank markets in an extended framework of Diamond and Dybvig (1983) where banks have idiosyncratic *preference shocks* of depositors. Contagion stems from contractual links between banks; for example, a lender bank loses deposits (and thus capital) in other banks, which causes an *insolvency* problem of the lender bank itself. Unlike these earlier contributions, our paper models the shocks on the asset side, giving endogenous illiquidity risk and giving rise to the interbank market, and highlights contagions through the impact on the illiquidity risk of financial institutions rather than the insolvency risk. Our model of the interbank market is a Walrasian economy, not a game; the actual agents (banks) are infinitesimal price-takers, not strategic players.

Our paper is close in spirit to the work of Diamond and Rajan (2005) in studying aggregate liquidity and banking crises. Both papers demonstrate that contagions arise from a shrinking of the pool of available aggregate liquidity and that the shock is on the asset side. We view our work as an addition to Diamond and Rajan (2005) with some key differences. First, the illiquidity risk in our model is endogenous, which stems from the (negative) shock on the asset quality. In Diamond and Rajan, there is no uncertainty of the asset quality; the shock is that a number of projects are *delayed* for exogenous reasons in generating cash flows, so the illiquidity risk is due to the need of some investors to consume early while some projects are delayed. Second, we model the two-way feedback between the interbank market run and creditor runs, highlighting the amplification effect of coordination failures and self-fulfilling beliefs. The interbank run in our model arises from precautionary liquidity-hoarding motives. Third, on the methodology side, we use global games to model creditor runs in a general equilibrium framework. This allows us to determine the unique equilibrium of a creditor run, and derive the unique interbank rate, and thus study the ex ante competitive equilibrium (the *laissez-faire* equilibrium).

Martin, Skeie and von Thadden (2013, 2014) also study runs on short-term funding markets. Their paper (2013) deals with the case of perfect markets for distressed assets, where they show how market fragility due to sudden collective expectation changes can be addressed by individual banks and regulators. Their paper (2014) focuses on how the different market microstructures (e.g., bilateral and tri-party repo) influence expectation-based runs. Their papers and our paper are complementary. In our model, banking crises originate in fundamental shocks and runs are both fundamental-based and expectation (panic)-based. We show the interplay between illiquidity

risk and insolvency risk in a financial system, with two-way feedback between the two types of runs.

A strand of literature studies portfolio choice of banks and how banks determine their level of liquidity by taking account of the opportunity of future fire sales of other banks (e.g., Allen and Gale (2004a,b), Gorton and Huang (2004), and Acharya et al. (2009), Bolton et al. (2011), Diamond and Rajan (2011), Gale and Yorulmazer (2013)).[30] Our paper differs and adds to this branch of research in various aspects. First, our paper studies the interbank lending market rather than the asset sale market. Second, the liquidity shock of banks in our paper is endogenous, originating in asset quality. Third, we model the feedback between the interbank market run and creditor runs, and examine the spiraling and amplification effect.[31] All papers in this literature, including ours, build on the result in Geanakoplos and Polemarchakis (1985) that when markets are incomplete, the competitive equilibrium may not be constrained efficient - a pecuniary externalities effect.[32]

7 Conclusion

This paper presents a theory of interbank lending; our model helps explain a systemic crisis. On the methodology side, we study global games in general equilibrium, so that we can explicitly model a *competitive* interbank market, and examine how the interaction among creditors within a bank affects and is affected by other banks through the interbank market.

Our model demonstrates banking crises originating in fundamental shocks, i.e., aggregate shocks on banks' asset quality, and shows the interplay between illiquidity risk and insolvency risk in a financial system context. The paper highlights the amplification mechanism of the feedback between the interbank market run and creditor runs, which can magnify a small shock to a systemic crisis.

---

[30] This literature is related to Shleifer and Vishny (1992) and Allen and Gale (1994, 1998) who show "cash-in-the-market prices".

[31] Our paper, in general, is also related to papers on the interaction between market and funding liquidity (Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009)), optimal maturity structure choices (Brunnermeier and Oehmke (2013)), the shadow banking system (Plantin (2013)), the effect of transparency on rollover risk (Bouvard, Chaigneau and De Motta (2013)), and the interaction between two runs (Goldstein (2005) and Liu (2011)).

[32] The work of Acharya and Skeie (2011) studies precautionary demand for liquidity of leveraged banks in inter-bank markets. The mechanism in their paper is different from ours. In theirs, precautionary hoarding of liquidity is to reduce the leverage and thus to commit to not conducting risk-shifting.

Financial contagion in our model occurs because institutions fish short-term funding from the same pool.

The model implies that the development of a well-functioning interbank market (for example, the global financial integration and openness in recent years) can improve the social welfare, i.e., this is because of both ex post better liquidity sharing and hence ex ante more efficient portfolio allocations. But it also contributes to a severe systemic crisis when an adverse shock hits, because of the contagion and amplification effects.
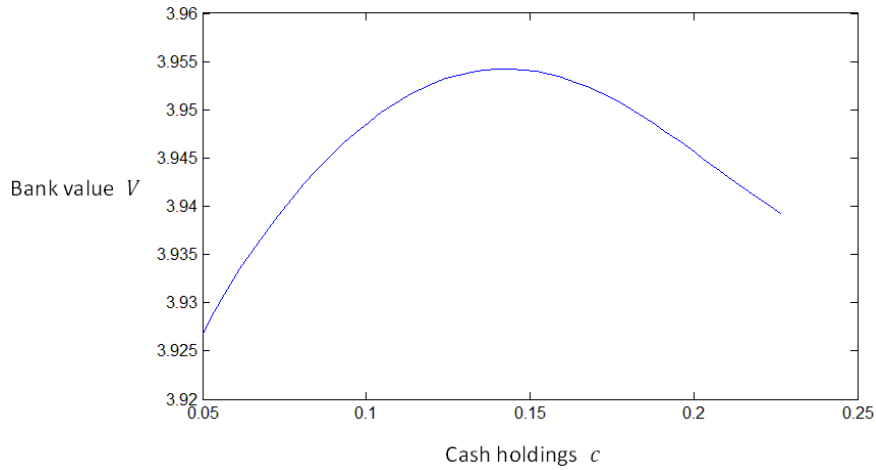
8 Appendix

A numerical example:

We provide a numerical example to illustrate the main results of the model. The numerical example is to show the qualitative (rather than quantitative) aspect of the model.

We set the parameter values as follows: $X = 8$, $F = 0.3$, $R_0 = 1.03$, $\frac{w_0}{\gamma} = 1$, $\underline{\pi} = 0.2$, $\overline{\pi} = 1$, $\mu = 0.5$, $\sigma = 1$, $\delta = 0.1$, $\mu_\pi = 0.5$, $\sigma_\pi = 1$. The endogenous variables in the model are $(c, R, \theta^*, \widehat{\theta}, I)$.

The second best equilibrium   The aggregate bank value V in Lemma 4 is a '∩'-shape function, as shown by Figure A-1. The optimal level of cash holdings is $c = 0.1420$, at which $V = 3.9542$, $R = 1.5417$, $\theta^* = 0.6552$, $\widehat{\theta} = -0.0005$, $\pi(\widehat{\theta}) = 0.3337$, and $I = 14.4978$.

Figure A-1: Aggregate bank value in the second best equilibrium

The competitive equilibrium   The equilibrium cash holdings for each individual bank are $c = 0.2265$, at which $V = 3.9395$, $R = 1.2466$, $\theta^* = 1.1933$, $\widehat{\theta} = -\infty$, and $\pi(\widehat{\theta}) = 0.2$. The interbank rate is $I = \frac{\underline{\pi} + \overline{\pi}}{2} X = 4.8$.
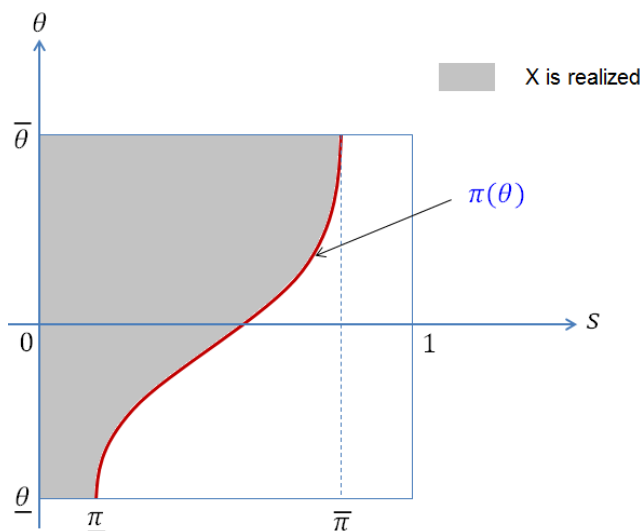
The amplification effect   Let the original allocation be under the competitive equilibrium for the normal state, that is, $c_N = 0.2265$ and $R_N = 1.2466$. When the extreme state occurs, it is assumed that $\mu_\varepsilon = -0.5$, in comparison with $\mu = 0.5$ in the normal state. In the new equilibrium, we can find that $\theta^*_{N\varepsilon} = 1.2033$, $\widehat{\theta}_{N\varepsilon} = 0.4007$, $\pi(\widehat{\theta}_{N\varepsilon}) = 0.5684$, and $I_{N\varepsilon} = 47.8506$. If agents

could foresee the adverse aggregate shock, the allocation under the competitive equilibrium would be $c_\varepsilon = 0.2452$ and $R_\varepsilon = 1.3816$.

Setup in Section 2.1:

The assumption of correlated payoffs across projects is very realistic, for example, as argued in Holmstrom and Tirole (1997). Here we provide the specific setup for the correlated asset payoffs across banks.

At T_2, the economy has a continuum of states, corresponding to the interval $[0, 1]$. The realization of the state will be drawn by the nature at T_2 according to the uniform probability distribution rule. That is, $s \sim U[0, 1]$, where s denotes the state of the economy. There exists a correspondence between asset quality at T_1 and asset payoffs at T_2. Specifically, denoting the payoff at T_2 in state s for an asset of quality θ at T_1 by function $\widetilde{x}(\theta, s)$, we have $\widetilde{x}(\theta, s) = \begin{cases} X & \text{if } s \in [0, \pi(\theta)] \\ 0 & \text{if } s \notin [0, \pi(\theta)] \end{cases}$.

Intuitively, an asset with higher quality is more 'robust' in the sense that it delivers cash flow X in more states. This characterization has two implications. First, for an asset of quality θ at T_1, the probability for it to realize X at T_2 perceived at T_1 is $\pi(\theta)$. Second, for a given realization of the state of the economy at T_2, if the quality-θ' asset pays 0, the quality-θ'' asset must pay 0 for any $\theta'' < \theta'$. The second implication means that the asset payoff realizations at T_2 across banks are correlated and, in particular, the risk of its debt cannot be reduced when a higher-quality bank acquires some assets from lower-quality banks. Figure A-2 illustrates correlated asset payoffs.

Figure A-2: Correlated asset payoffs

By the second implication above, the cash flow pattern at T_2 for a lender bank in the interbank market is $\begin{cases} (1-c)X + \Delta \\ 0 \end{cases}$, where Δ is the "interest income" from the interbank lending. So staying creditors in this bank cannot gain from Δ because the debt contract is a "fixed-income" security and $(1-c)X$ is sufficient to cover the creditors' notional claim by noting (10). If, instead, the asset payoffs across banks are (perfectly) uncorrected, the cash flow pattern at T_2 for a lender bank is $\begin{cases} (1-c)X \\ \Delta \end{cases}$. In this case, staying creditors benefit from their bank's interbank lending; a higher I and hence higher Δ make their debt less risky.
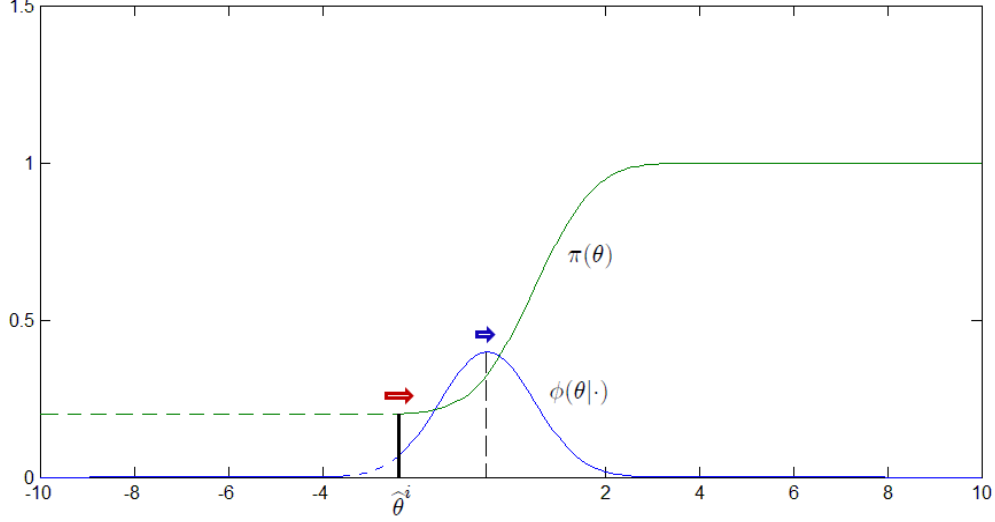
Proof of Lemma 1:

The prior density of θ is $g(\theta)$ with distribution $N(\mu, \sigma^2)$. The signal is $\theta^* = \theta + \delta\epsilon$, where $\epsilon \sim N(0,1)$. So, the posterior density $h_g(\theta|\theta^*)$ is with distribution $N(\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*, \frac{1}{\alpha+\beta})$, where $\alpha = \frac{1}{\sigma^2}$ and $\beta = \frac{1}{\delta^2}$. We can write $h_g(\theta|\theta^*) = \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*, \sqrt{\frac{1}{\alpha+\beta}})$.

By (2), we have $\widehat{\theta}^i = \theta^* - \delta\Phi^{-1}(\frac{c}{F})$. So we can combine (1) and (2):

$$\int_{\theta=\theta^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*, \sqrt{\frac{1}{\alpha+\beta}}) d\theta = \frac{w_0}{\gamma}. \tag{A1}$$

Denote the LHS of (A1) by $Y(\theta^*)$, which is a function of θ^*.

The key to the proof is to determine monotonicity of $Y(\theta^*)$. There are three forces in determining the monotonicity, as shown in Figure A-3: i) The lower boundary of the integral, $\theta^* - \delta\Phi^{-1}(\frac{c}{F})$, is increasing in θ^*, so $Y(\theta^*)$ tends to be decreasing in θ^*; ii) The conditional mean, $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*$, is increasing in θ^*, so $Y(\theta^*)$ tends to be increasing in θ^*; iii) $\pi(\theta)$ is an increasing function, which makes $Y(\theta^*)$ tend to be increasing in θ^*.

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Figure A-3: The forces in determining the monotonicity of $Y(\theta^*)$

First, we consider the limiting case of $\frac{\delta}{\sigma} \to 0$. In this case, $\frac{\beta}{\alpha+\beta} \to 1$. Hence, the conditional mean, $\frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*$, and the lower boundary of the integral, $\theta^* - \delta\Phi^{-1}(\frac{c}{F})$, increase with the *same* speed. So $Y(\theta^*)$ is certainly increasing since $\pi(\theta)$ is an increasing function. Concretely, under the limit $\delta \to 0$ for a given σ, we have:

$$
\begin{aligned}
Y(\theta^*) &= R \cdot \pi(\theta^*) \cdot \int_{\theta^* - \delta\Phi^{-1}(\frac{c}{F})}^{+\infty} \phi(\theta; \theta^*, \delta)d\theta \\
&= R \cdot \pi(\theta^*) \cdot \Phi\left(\frac{\theta^* - (\theta^* - \delta\Phi^{-1}(\frac{c}{F}))}{\delta}\right) \\
&= R \cdot \pi(\theta^*) \cdot \frac{c}{F}.
\end{aligned}
$$

So (3) is obtained. (A1) admits a unique solution with respect to θ^* (under proper parametric values of c, F and R) because $Y(\theta^*)$ is monotonically increasing. Similarly, we can prove the case for the limit $\sigma \to +\infty$ for a given δ.

Next, we consider the non-limiting case. We show that when δ is small but not zero for a given
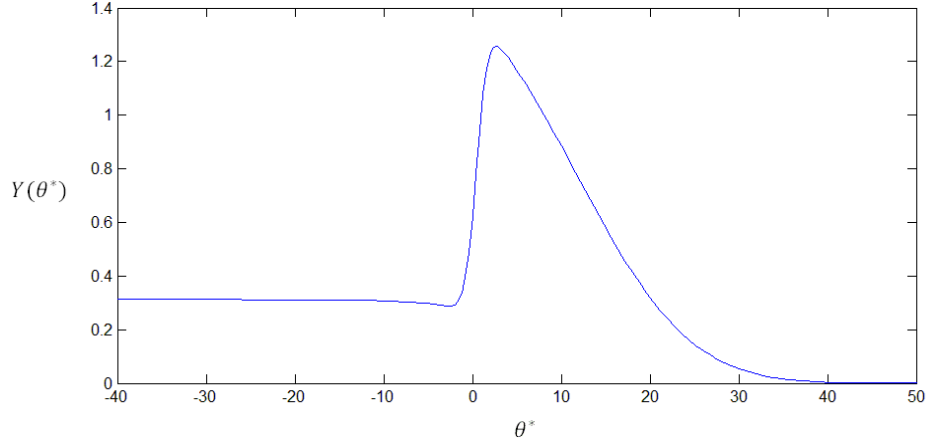
σ, (A1) can admit two solutions. Concretely, under this condition, $Y(\theta^*)$ becomes

$$
\begin{aligned}
Y(\theta^*) &= \int_{\theta=\theta^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R\cdot\pi(\theta)\cdot\phi(\theta;\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*,\sqrt{\frac{1}{\alpha+\beta}})d\theta \\
&= R\cdot\pi(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*)\cdot\Phi(\frac{(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*)-(\theta^*-\delta\Phi^{-1}(\frac{c}{F}))}{\sqrt{\frac{1}{\alpha+\beta}}}) \\
&= R\cdot\pi(\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*)\cdot\Phi(\frac{\frac{\alpha}{\alpha+\beta}(\mu-\theta^*)+\delta\Phi^{-1}(\frac{c}{F})}{\sqrt{\frac{1}{\alpha+\beta}}}) \qquad\text{(A2)}
\end{aligned}
$$

The second term in the third line above reflects the effect of $\pi(\theta)$ being an increasing function. The third term captures the effect that the conditional mean increases slower than the lower boundary of the integral. The second term is increasing in θ^* *but very slowly when θ^* is very low or very high* (as shown in Figure A-3), and the third term is decreasing in θ^*. So, whether $Y(\theta^*)$ is increasing in θ^* is ambiguous. The first order derivative on $Y(\theta^*)$ is

$$
\begin{aligned}
\frac{\partial Y(\theta^*)}{\partial\theta^*} &\propto \left[\underline{\pi}+(\overline{\pi}-\underline{\pi})\Phi(\frac{\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*-\mu_\pi}{\sigma_\pi})\right]\cdot\phi(\frac{\frac{\alpha}{\alpha+\beta}(\mu-\theta^*)+\delta\Phi^{-1}(\frac{c}{F})}{(\frac{1}{\alpha+\beta})^{\frac{1}{2}}})(-\frac{\frac{\alpha}{\alpha+\beta}}{(\frac{1}{\alpha+\beta})^{\frac{1}{2}}}) \\
&\quad+(\overline{\pi}-\underline{\pi})\phi(\frac{\frac{\alpha}{\alpha+\beta}\mu+\frac{\beta}{\alpha+\beta}\theta^*-\mu_\pi}{\sigma_\pi})(\frac{\frac{\beta}{\alpha+\beta}}{\sigma_\pi})\cdot\Phi(\frac{\frac{\alpha}{\alpha+\beta}(\mu-\theta^*)+\delta\Phi^{-1}(\frac{c}{F})}{(\frac{1}{\alpha+\beta})^{\frac{1}{2}}})
\end{aligned}
$$

Hence, there are three regions for $Y(\theta^*)$. When θ^* is very low, $\frac{\partial Y(\theta^*)}{\partial\theta^*}<0$. In the middle region around $\theta^*=\mu_\pi$ (where the second term in (A2) is increasing fast in θ^*), $\frac{\partial Y(\theta^*)}{\partial\theta^*}>0$. When θ^* is very high, $\frac{\partial Y(\theta^*)}{\partial\theta^*}<0$ again. Figure A-4 plots $Y(\theta^*)$ for a set of parameters, where $F=0.6, c=0.26$, $R=1.5598$, $\underline{\pi}=0.2$, $\overline{\pi}=1$, $\mu=\mu_\pi=0.5$, $\sigma=\sigma_\pi=1, \delta=0.1$. If $\frac{w_0}{\gamma}=1$, the two solutions are $\theta^*=0.9595$ and $\theta^*=8.086$, which correspond to $\pi(\theta^*=0.9595)=0.7416$ and $\pi(\theta^*=8.086)=1$. Clearly, the higher solution is close to being the "bad" equilibrium, i.e., creditors run on the bank no matter what signals they receive.

Figure A-4: The case of non-monotonicity of $Y(\theta^*)$

Therefore, when δ is small but not zero for a given σ, (A1) can admit two solutions (under proper parametric values of c, F and R): one is around $\theta^* = \mu_\pi$ and the other is higher.[33] We prove that the lower solution corresponds to a stable equilibrium while the higher solution corresponds to an unstable equilibrium. In a stable equilibrium, the best response function (of an individual creditor to its peers) intersects the 45 degree line at a slope of less than 1. Let an individual creditor's threshold be θ^{i*} and its peers' be θ^*. So

$$\int\limits_{\theta=\theta^*-\delta\Phi^{-1}(\frac{c}{F})}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^{i*}, \sqrt{\frac{1}{\alpha+\beta}})d\theta = \frac{w_0}{\gamma}$$

We write the LHS as the function $Y(\theta^*, \theta^{i*})$. In solving the equation $Y(\theta^*, \theta^{i*}) = \frac{w_0}{\gamma}$, we have $\frac{\partial \theta^{i*}}{\partial \theta^*} = -\frac{\frac{\partial Y}{\partial \theta^*}}{\frac{\partial Y}{\partial \theta^{i*}}}$. Based on the previous result, Y is an increasing function at the lower solution, that is, $Y(\theta^* + \triangle, \theta^{i*} + \triangle) - Y(\theta^*, \theta^{i*}) > 0$ for a small positive $\triangle$. So $(\frac{\partial Y}{\partial \theta^*} + \frac{\partial Y}{\partial \theta^{i*}})\triangle > 0$, and thus $\frac{\partial Y}{\partial \theta^{i*}} > -\frac{\partial Y}{\partial \theta^*} > 0$. Therefore, $\frac{\partial \theta^{i*}}{\partial \theta^*} < 1$. Similarly, we can prove that $\frac{\partial \theta^{i*}}{\partial \theta^*} > 1$ at the higher solution.

If the creditor-run game has two threshold equilibria, it is reasonable to select the equilibrium corresponding to the lower solution of θ^* under the following three criteria: i) It is a stable equilibrium. Stability is the equilibrium selection criterion in Frankel, Morris and Pauzner (2003), Vives (2005, 2013)), Angeletos, Hellwig and Pavan (2007), etc.; ii) It Pareto-dominates the other equilibrium; iii) The other (higher) solution becomes extremely large and approaches infinite (i.e., $\theta^* \to +\infty$) when $\frac{\delta}{\sigma} \to 0$, which is thus close to being the bad equilibrium (i.e., creditors run on the bank no matter what signals they receive).

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[33]In the low region of θ^*, $\pi(\theta)$ is low, so (A1) does not have solutions.

Finally, we conduct comparative statics. We can write the solution to (1) as the reaction function $\theta^*(\widehat{\theta}^i; R)$ and the solution to (2) as the reaction function $\widehat{\theta}^i(\theta^*; c)$. It is easy to show that $\frac{\partial \widehat{\theta}^i}{\partial c} < 0$ and $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} > 0$ in (2), and $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ and $\frac{\partial \theta^*}{\partial R} < 0$ in (1). Hence, a decrease in c leads to θ^* increasing along an upward spiral (feedback) in that $\frac{\partial \widehat{\theta}^i}{\partial c} < 0$ in (2), $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ in (1), and $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} > 0$ in (2). Similarly, a decrease in R leads to θ^* increasing along an upward spiral (feedback) in that $\frac{\partial \theta^*}{\partial R} < 0$ in (1), $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} > 0$ in (2) , and $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ in (1). Also, we can show $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} < 1$ in (1). In fact, if we write the LHS of (1) as the function $Y(\theta^*, \widehat{\theta}^i)$, we see $Y(\theta^* + \triangle, \widehat{\theta}^i + \triangle) - Y(\theta^*, \widehat{\theta}^i) > 0$ in the previous proof.

Proof of Lemma 2:

First, we find conditions under which the bank survives if and only if its asset quality is above a threshold. We find a sufficient condition – the condition under which the function $LS(\theta)$ is increasing in θ, where $LS(\theta)$ is defined as

$$LS(\theta) = \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)\right] \pi(\theta)}{I} - \left[F \cdot H(\frac{\theta^* - \theta}{\delta}) - c\right]$$

The first order derivative of $LS(\theta)$ is

$$\frac{\partial LS(\theta)}{\partial \theta} = \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta})\right)\right] \pi'(\theta)}{I} + \frac{F}{\delta} \cdot h(\frac{\theta^* - \theta}{\delta}) \left[1 - \frac{R \cdot \pi(\theta)}{I}\right]$$

Therefore, a sufficient condition for $\frac{\partial LS(\theta)}{\partial \theta} > 0$ is $I > R$ by noting that $\pi(\theta) \leq 1$ for any θ.

We start the formal proof. The proof is similar to the proof of Lemma 1. First, we obtain $\widehat{\theta}^i$ as a function of θ^* in (5), denoted by $\widehat{\theta}^i(\theta^*)$. We prove that under a weak condition it follows $0 \leq \frac{d\widehat{\theta}^i(\theta^*)}{d\theta^*} < 1$. That is, $\widehat{\theta}^i$ is increasing in θ^*, but the speed of the increase in $\widehat{\theta}^i$ is lower than the speed of θ^*. This is clearly true if $\widehat{\theta}^i$ is the corner solution in (5). We focus on the case of non-corner solutions in (5). By differentiating on both sides of (5a), we can obtain

$$\frac{\partial \widehat{\theta}^i}{\partial \theta^*} = \frac{F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta}) - \frac{R \cdot F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta})\pi(\widehat{\theta}^i)}{I}}{\left[\frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \widehat{\theta}^i}{\delta})\right)\right]\pi'(\widehat{\theta}^i)}{I} + \frac{R \cdot F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(-\frac{1}{\delta})\pi(\widehat{\theta}^i)}{I}\right] + F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta})}$$

$$= \frac{F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta})(1 - \frac{R \cdot \pi(\widehat{\theta}^i)}{I})}{\frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \widehat{\theta}^i}{\delta})\right)\right]\pi'(\widehat{\theta}^i)}{I} + F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta})(1 - \frac{R \cdot \pi(\widehat{\theta}^i)}{I})}$$

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So, if $R \cdot \pi(\widehat{\theta}^i) < I$ and $I \neq +\infty$, we have that $0 < \frac{d\widehat{\theta}^i(\theta^*)}{d\theta^*} < 1$. Also, if $R < I$, it must be the case that $R \cdot \pi(\widehat{\theta}^i) < I$ by noting that $\pi(\theta) \leq 1$ for any θ. Therefore, a sufficient condition to guarantee $0 < \frac{d\widehat{\theta}^i(\theta^*)}{d\theta^*} < 1$ is $R < I$ and $I \neq +\infty$.

Substituting $\widehat{\theta}^i(\theta^*)$ into (4), we have

$$
\int\limits_{\theta=\widehat{\theta}^i(\theta^*)}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*, \sqrt{\frac{1}{\alpha+\beta}})d\theta = \frac{w_0}{\gamma}. \tag{A3}
$$

Denote LHS of (A3) as $Y(\theta^*)$. By the proof of Lemma 1, if $\widehat{\theta}^i$ increases as fast as θ^*, $Y(\theta^*)$ is increasing in θ^*. Now because $\widehat{\theta}^i$ increases less faster than θ^*, $Y(\theta^*)$ is certainly increasing in θ^*.

Concretely, by $\frac{d\widehat{\theta}^i(\theta^*)}{d\theta^*} < 1$, we have that $\widehat{\theta}^i(\theta^* + \triangle) < \widehat{\theta}^i(\theta^*) + \triangle$ for a small positive $\triangle$. Therefore,

$$
\begin{aligned}
Y(\theta^* + \triangle) &= \int\limits_{\theta=\widehat{\theta}^i(\theta^*+\triangle)}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}(\theta^* + \triangle), \sqrt{\frac{1}{\alpha+\beta}})d\theta \\
&> \int\limits_{\theta=\widehat{\theta}^i(\theta^*)+\triangle}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}(\theta^* + \triangle), \sqrt{\frac{1}{\alpha+\beta}})d\theta \\
&\geq \int\limits_{\theta=\widehat{\theta}^i(\theta^*)}^{\theta=+\infty} R \cdot \pi(\theta) \cdot \phi(\theta; \frac{\alpha}{\alpha+\beta}\mu + \frac{\beta}{\alpha+\beta}\theta^*, \sqrt{\frac{1}{\alpha+\beta}})d\theta \\
&= Y(\theta^*)
\end{aligned}
$$

The second inequality above is proved in the proof of Lemma 1 when $\frac{\delta}{\sigma} \to 0$, or for the middle region of θ^* (around $\theta^* = \mu_\pi$) when δ is small but not zero for a given σ.

To summarize, a sufficient condition for the creditor-run game having a unique (stable) equilibrium is $R < I$ and $I \neq +\infty$ when δ is small but not zero for a given δ. A wide range of parameter values (e.g., X is big enough) can guarantee that $R < I$ in equilibrium under any c. The numerical example above in the Appendix illustrates one case under a set of parameter values.

Finally, we conduct comparative statics. We have already proved that $\frac{\partial \widehat{\theta}^i}{\partial \theta^*} \geq 0$ in (5). Now we prove that $\frac{\partial \widehat{\theta}^i}{\partial I} \geq 0$ in (5) and $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ in (4). The result of $\frac{\partial \theta^*}{\partial \widehat{\theta}^i} > 0$ in (4) can be immediately

obtained, which is the same as the case in (1). By differentiating on (5a), we can obtain

$$\frac{\partial \widehat{\theta}^i}{\partial I} = \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \widehat{\theta}^i}{\delta})\right)\right] \pi(\widehat{\theta}^i)(\frac{1}{I^2})}{F \cdot h(\frac{\theta^* - \widehat{\theta}^i}{\delta})(\frac{1}{\delta})(1 - \frac{R \cdot \pi(\widehat{\theta}^i)}{I}) + \left[\frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \widehat{\theta}^i}{\delta})\right)\right] \pi'(\widehat{\theta}^i)}{I}\right]}$$

Under the sufficient condition $R < I$, we have that $1 - \frac{R \cdot \pi(\widehat{\theta}^i)}{I} > 0$ and thus that $\frac{\partial \widehat{\theta}^i}{\partial I} > 0$.

Proof of Proposition 1:

With an interbank market, the equilibrium at T_1 is characterized by the triplet $(\theta^*, \widehat{\theta}, I)$, which solves the system of equations (4)-(5), (6) and (7) for given c and R.

By (6), we can replace $\widehat{\theta}^i$ in (4)-(5) with $\widehat{\theta}$. By substituting the corner solution $\widehat{\theta} = \underline{\theta}$ into (4), we can solve the unique θ^* for a given R, denoted as $\theta^{*T}(R)$. Then we can substitute $\theta^* = \theta^{*T}(R)$ and $\widehat{\theta} = \underline{\theta}$ into (7), we can find out the following cutoff value of c (for a given R):

$$c^T(R) = \int_{\underline{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^{*T}(R) - \theta}{\delta}) g(\theta) d\theta$$

Now we can divide c into three ranges: $c < c^T(R)$, $c = c^T(R)$ and $c > c^T(R)$.

In the first range of $c < c^T(R)$, the solution of $(\theta^*, \widehat{\theta}, I)$ is non-corner one. Concretely, we can find the solution in the following way. First, based on Lemma 2, we obtain the unique solution of pair $(\theta^*, \widehat{\theta})$ for a given I. Denote the solution by $\theta^*(I)$ and $\widehat{\theta}(I)$. Then, by substituting $\theta^*(I)$ and $\widehat{\theta}(I)$ into the market clearing condition (7), we can find out I, that is,

$$\int_{\widehat{\theta}(I)}^{\overline{\theta}} cg(\theta) d\theta = \int_{\widehat{\theta}(I)}^{\overline{\theta}} F \cdot H(\frac{\theta^*(I) - \theta}{\delta}) g(\theta) d\theta$$

In the second range of $c = c^T(R)$, we have that $\widehat{\theta} = \underline{\theta}$ and $\theta^* = \theta^{*T}(R)$ in equilibrium. There is more than one interbank rate I that can clear the interbank market. The equilibrium interbank rate can be found in the following way. We first work out the maximum rate that bank $\underline{\theta}$ can afford (where $\underline{\theta} = -\infty$):

$$\begin{aligned} I(\widehat{\theta} &= \underline{\theta}) = \frac{\left[(1-c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \underline{\theta}}{\delta})\right)\right] \pi(\underline{\theta})}{F \cdot H(\frac{\theta^* - \underline{\theta}}{\delta}) - c} \\ &= \frac{[(1-c)X] \pi}{F - c} \end{aligned}$$

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Then, any rate $I \in [1, I(\underline{\theta})]$ can be the equilibrium rate. The equilibrium rate can be lower than $I(\underline{\theta})$ because no banks of even lower quality compete with banks of quality $\underline{\theta}$. Hence, although the interbank rate falls below $I(\underline{\theta})$, the aggregate demand for liquidity does not change and thus the interbank market is still cleared.

In the third range of $c > c^T(R)$, the aggregate supply of liquidity exceeds the maximum aggregate demand. So the equilibrium rate must be equal to 1, which is the return of storing cash.

Proof of Lemma 3 and Lemma 4:

We illustrate that the aggregate value in the objective function of Program 1 is equal to the sum of the equityholder's payoff across banks in (8) plus the sum of debtholders' payoffs across banks on RHS of (9'). By (8), the aggregate equity value is

$$\int_{\widehat{\theta}}^{\widehat{\overline{\theta}}} \left\{ \left[c - F \cdot H(\frac{\theta^* - \theta}{\delta}) \right] I + \left[(1 - c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \right] \pi(\theta) \right\} g(\theta)d\theta$$

$$= \int_{\widehat{\theta}}^{\overline{\theta}} \left[(1 - c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \right] \pi(\theta) \cdot g(\theta)d\theta$$

The second line in the above is obtained because the first term in the first line is cancelled in aggregating by (7), i.e., $\int_{\widehat{\theta}}^{\overline{\theta}} cg(\theta)d\theta = \int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^* - \theta}{\delta})g(\theta)d\theta$. Intuitively, the gains and losses in the interbank market across banks cancel out. By (9'), the aggregate debt value is

$$\int_{\underline{\theta}}^{\widehat{\theta}} c \cdot g(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[F \cdot H(\frac{\theta^* - \theta}{\delta}) + R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \cdot \pi(\theta) \right] g(\theta)d\theta$$

Therefore, the aggregate value in the economy is

$$\int_{\widehat{\theta}}^{\overline{\theta}} \left[(1 - c)X - R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \right] \pi(\theta) \cdot g(\theta)d\theta$$

$$+ \int_{\underline{\theta}}^{\widehat{\theta}} c \cdot g(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[F \cdot H(\frac{\theta^* - \theta}{\delta}) + R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \cdot \pi(\theta) \right] g(\theta)d\theta$$

$$= \int_{\underline{\theta}}^{\widehat{\theta}} c \cdot g(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[(1 - c)(X \cdot \pi(\theta)) + F \cdot H(\frac{\theta^* - \theta}{\delta}) \right] g(\theta)d\theta$$

$$= \int_{\underline{\theta}}^{\widehat{\theta}} c \cdot g(\theta)d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} \left[c + (1 - c)(X \cdot \pi(\theta)) \right] g(\theta)d\theta$$

The last line in the above is obtained by using (7) again.

We turn to Lemmas 3 and 4. First, for a given c, the system of equations (4)-(5), (6), (7) and (9') determines the vector $(R, \widehat{\theta}, \theta^*, I)$. In fact, based on Proposition 1, for given c and R, the pair

$(\theta^*, \widehat{\theta})$ is determined. Plugging the functions of $\theta^*(c, R)$ and $\widehat{\theta}(c, R)$ into (9'), we can solve the equation of (9') with respect to R. Overall, we can find $\widehat{\theta}$ as a function of c.

Second, by plugging the function of $\widehat{\theta}(c)$ into the bank value (denoted by V) in the objective function in Program 1, we obtain V as a function of c. Because all functions are continuous, the optimization of V on the close set $[0, 1]$ clearly has solutions.

Proof of Lemma 5:

Denote the equity value in (11) by V_E. The first order derivative on V_E is

$$
\begin{aligned}
\frac{\partial V_E}{\partial c^i} &= \int_{\widehat{\theta}^i(c^i)}^{\overline{\theta}} [I - X \cdot \pi(\theta)] \, g(\theta) d\theta \\
&\quad - \frac{\partial \widehat{\theta}^i(c^i)}{\partial c^i} \cdot \left\{ \left[\left[c^i - F \cdot H(\frac{\theta^* - \theta}{\delta}) \right] I + \left[\begin{array}{c} (1 - c^i)X - \\ R \cdot F \cdot \left(1 - H(\frac{\theta^* - \theta}{\delta}) \right) \end{array} \right] \pi(\theta) \right] g(\theta) \right\} \Big|_{\theta = \widehat{\theta}^i(c^i)} \\
&= \int_{\widehat{\theta}^i(c^i)}^{\overline{\theta}} [I - X \cdot \pi(\theta)] \, g(\theta) d\theta
\end{aligned}
$$

Our proof has three steps. First, we prove that if the competitive equilibrium exists, it must be true that $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$ for any c^i. We prove by contradiction. If $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$ is not true, we have that $\frac{\partial \widehat{\theta}^i(c^i)}{\partial c^i} < 0$ for sufficiently small c^i, by noting that $\widehat{\theta}^i(c^i) = \max(\underline{\theta}, \theta^T(c^i))$ and $\frac{\partial \theta^T(c^i)}{\partial c^i} < 0$. This would mean that the optimum c^i for an individual bank is either 0 or 1, which would in turn mean that the fixed point of $c \to (R, \widehat{\theta}, \theta^*, I) \to c$ does not exist. In fact, there are three possible cases for $\frac{\partial V_E}{\partial c^i}$: i) $I - X \cdot \pi(\overline{\theta}) \geq 0$. In the first case, $\frac{\partial V_E}{\partial c^i} > 0$ for any c^i. So V_E is strictly increasing in c^i. ii) $I - X \cdot \pi(\underline{\theta}) \leq 0$. In the second case, $\frac{\partial V_E}{\partial c^i} < 0$ for any c^i. So V_E is strictly decreasing in c^i. iii) $I - X \cdot \pi(\overline{\theta}) < 0 < I - X \cdot \pi(\underline{\theta})$. In the third case, if $\frac{\partial V_E}{\partial c^i}|_{c^i = 0} > 0$, then $\frac{\partial V_E}{\partial c^i} > 0$ for any c^i; if $\frac{\partial V_E}{\partial c^i}|_{c^i = 0} < 0$, then $\frac{\partial V_E}{\partial c^i} < 0$ when c^i is small, so V_E is decreasing in c^i when c^i is small and then either decreases further or starts to increase when c^i increases. Overall, in the above three cases, the optimum c^i to maximize V_E is either 0 or 1. The driving force behind these extreme decisions is that an individual bank takes I as given and being constant, with the outcome of the "constant returns to scale" effect.

Alternatively, we prove that if $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$ is not true, it is impossible to *simultaneously* have $\frac{\partial V_E}{\partial c^i} = 0$ and $\frac{\partial^2 V_E}{\partial (c^i)^2} < 0$ (i.e., for maximum not minimum). Under $\frac{\partial V_E}{\partial c^i} = 0$, the third case above must be true, that is, $I - X \cdot \pi(\overline{\theta}) < 0 < I - X \cdot \pi(\underline{\theta})$ and $\int_{\widehat{\theta}^i(c^i)}^{\overline{\theta}} [I - X \cdot \pi(\theta)] \, g(\theta) d\theta = 0$. Now

if c^i is increased a bit and thus $\widehat{\theta}^i(c^i)$ is decreased a bit, we have $\int_{\widehat{\theta}^i(c^i)}^{\overline{\theta}} [I - X \cdot \pi(\theta)] \, g(\theta)d\theta > 0$ by noting $I - X \cdot \pi(\theta)$ is decreasing in θ. So $\frac{\partial^2 V_E}{\partial (c^i)^2} > 0$. In other words, at the optimum where $\frac{\partial V_E}{\partial c^i} = 0$, it achieves minimum rather than maximum.

Second, we prove that under conditions i) $I = \int_{\underline{\theta}}^{\overline{\theta}} [X \cdot \pi(\theta)] \, g(\theta)d\theta$ and ii) $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$, the competitive equilibrium exists. Condition ii) implies $\widehat{\theta}^i(c^i = 0) = \underline{\theta}$, which can be written as $\frac{X\pi(\underline{\theta})}{I} \geq F$ (or $I \leq \frac{X\pi(\underline{\theta})}{F}$). We first examine the left part of the fixed point problem of $c \to (R, \widehat{\theta}, \theta^*, I) \to c$. Based on the proof of Proposition 1, if the level of cash holdings c is such that it exactly achieves the ex post efficiency (i.e., $\widehat{\theta} = \underline{\theta}$), any rate $I \in [1, I(\underline{\theta})]$ supports the equilibrium, where $I(\underline{\theta}) = \frac{[(1-c)X]\pi}{F-c}$. It is easy to prove that $\frac{X\pi(\underline{\theta})}{F} < \frac{[(1-c)X]\pi}{F-c}$. Consequently, under condition ii), the interbank rate I automatically satisfies $I \in [1, I(\underline{\theta})]$. Then, we go to the right part of the fixed point problem. Conditions i) and ii) together imply that $\frac{\partial V_E}{\partial c^i} \equiv 0$. So, an individual bank can choose any c^i as its optimal cash holdings, including the one that achieves the ex post efficiency. In sum, under conditions i) and ii), the fixed point problem has solutions. In order to guarantee that conditions i) and ii) are simultaneously satisfied, we need $\int_{\underline{\theta}}^{\overline{\theta}} [X \cdot \pi(\theta)] \, g(\theta)d\theta \leq \frac{X\pi(\underline{\theta})}{F}$, which means $F \leq \frac{2\pi}{\pi + \overline{\pi}}$.

Here we also provide the concrete procedure to work out the competitive equilibrium. The procedure can be summarized as follows: $\widehat{\theta} = -\infty \overset{(4) \,\&\, (9')}{\Longrightarrow} (\theta^*, R) \overset{(7)}{\Longrightarrow} c \overset{(5)}{\Longrightarrow} I(\widehat{\theta} = -\infty)$. First, by plugging $\widehat{\theta} = -\infty$ into (4) and (9'), we can find the pair (θ^*, R) by solving the system of equations. Then, by plugging $\widehat{\theta} = -\infty$ and the solved θ^* into (7), we can obtain the optimal c. Finally, through (5), we can work out the maximum interbank rate that the marginal bank can afford.

Last, we need to prove that the competitive equilibrium is unique, that is, the solution to the fixed point problem of $c \to (R, \widehat{\theta}, \theta^*, I) \to c$ is unique. In fact, based on the proof of Proposition 1, if the c is different from the one that achieves the ex post efficiency, we have that $\widehat{\theta} > \underline{\theta}$. Now if an individual bank deviates and increases cash a bit (i.e., $c^i > c$), this leads to $\underline{\theta} < \widehat{\theta}^i(c^i) < \widehat{\theta}$. That is, $\widehat{\theta}^i(c^i) \equiv \underline{\theta}$ is not true, and hence an individual bank's optimal c is either 1 or 0 as shown in the first step. So this cannot form a fixed point solution.

Proof of Proposition 2:

In the constrained second best equilibrium, $\widehat{\theta} \geq \underline{\theta}$, while in the competitive equilibrium, $\widehat{\theta} = \underline{\theta}$. Considering $\frac{\partial \widehat{\theta}}{\partial c} < 0$, the amount of cash holdings is (weakly) higher in the competitive equilibrium

than in the second best equilibrium.

Proof of Lemma 6:

First, we analyze (13a). We can rewrite (13a) as

$$c = \frac{\int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^* - \theta}{\delta}) g(\theta; \mu) d\theta}{\int_{\widehat{\theta}}^{\overline{\theta}} g(\theta; \mu) d\theta} \tag{A4}$$

Write RHS of (A4) as a function with respect to $\widehat{\theta}$ parameterized with θ^*, denoted by $AL(\widehat{\theta}; \theta^*)$. The first order derivative of $AL(\widehat{\theta}; \theta^*)$ is

$$\frac{\partial AL}{\partial \widehat{\theta}} = F \cdot g(\widehat{\theta}; \mu) \frac{-H(\frac{\theta^* - \widehat{\theta}}{\delta}) \int_{\widehat{\theta}}^{\overline{\theta}} g(\theta; \mu) d\theta + \int_{\widehat{\theta}}^{\overline{\theta}} H(\frac{\theta^* - \theta}{\delta}) g(\theta; \mu) d\theta}{\int_{\widehat{\theta}}^{\overline{\theta}} g(\theta; \mu) d\theta}$$

$$< 0$$

The inequality above is because $H(\frac{\theta^* - \theta}{\delta})$ is decreasing in θ. So, we have that $AL(\widehat{\theta}; \theta^*)$ is decreasing in $\widehat{\theta}$ (for a given θ^*), and it achieves its highest value at $\widehat{\theta} = \underline{\theta}$.

Therefore, when $c \leq AL(\widehat{\theta} = \underline{\theta}; \theta^*)$, the equation (13a) has a unique solution with respect to $\widehat{\theta}$ for a given θ^*. Denote the solution by $\widehat{\theta}(\theta^*; \mu)$.

We can do the comparative static analysis on the solution $\widehat{\theta}(\theta^*; \mu)$. We prove that $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 1$ and $\frac{\partial \widehat{\theta}}{\partial \mu} < 0$ under some sufficient conditions. We first prove the limiting case of $\delta \to 0$ for a given σ. In fact, under the limit $\delta \to 0$, we have the limiting function $H(\frac{\theta^* - \theta}{\delta}) = \begin{cases} 1 & \text{when } \theta < \theta^* \\ [0, 1] & \text{when } \theta = \theta^* \\ 0 & \text{when } \theta > \theta^* \end{cases}$.

Also, the function $H(\frac{\theta^* - \theta}{\delta})$ with respect to θ converges *almost uniformly* to the limiting function when $\delta \to 0$. Hence, (13a) can be transformed into

$$\int_{\widehat{\theta}}^{\overline{\theta}} c g(\theta; \mu) d\theta = \int_{\widehat{\theta}}^{\overline{\theta}} F \cdot H(\frac{\theta^* - \theta}{\delta}) g(\theta; \mu) d\theta$$

$$\Leftrightarrow \quad c \left(1 - G(\widehat{\theta}; \mu, \sigma)\right) = F \left(G(\theta^*; \mu, \sigma) - G(\widehat{\theta}; \mu, \sigma)\right)$$

$$\Leftrightarrow \quad c \left(1 - \Phi(\frac{\widehat{\theta} - \mu}{\sigma})\right) = F \left(\Phi(\frac{\theta^* - \mu}{\sigma}) - \Phi(\frac{\widehat{\theta} - \mu}{\sigma})\right)$$

$$\Leftrightarrow \quad \Phi(\frac{\widehat{\theta} - \mu}{\sigma}) = \frac{F \cdot \Phi(\frac{\theta^* - \mu}{\sigma}) - c}{F - c}$$

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So, $\Phi(\frac{\widehat{\theta}-\mu}{\sigma}) < \Phi(\frac{\theta^*-\mu}{\sigma})$ and hence $\widehat{\theta} < \theta^*$. We have the following comparative statics:

$$\frac{\partial \widehat{\theta}}{\partial \theta^*} = \frac{F}{F-c} \frac{\phi(\frac{\theta^*-\mu}{\sigma})}{\phi(\frac{\widehat{\theta}-\mu}{\sigma})}$$

$$\frac{\partial \widehat{\theta}}{\partial \mu} = \frac{-\frac{F}{F-c}\phi(\frac{\theta^*-\mu}{\sigma}) + \phi(\frac{\widehat{\theta}-\mu}{\sigma})}{\phi(\frac{\widehat{\theta}-\mu}{\sigma})}.$$

Clearly, $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 0$. We also prove that $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 1$ and $\frac{\partial \widehat{\theta}}{\partial \mu} < 0$ under some sufficient conditions. In fact, if $\frac{F}{F-c} \frac{\phi(\frac{\theta^*-\mu}{\sigma})}{\phi(\frac{\widehat{\theta}-\mu}{\sigma})} > 1$, which is true when c is sufficiently high (in which case $\widehat{\theta}$ is close to being $-\infty$ and hence $\phi(\frac{\widehat{\theta}-\mu}{\sigma})$ is close to being 0), we have that $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 1$ and $\frac{\partial \widehat{\theta}}{\partial \mu} < 0$.

The comparative results $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 1$ and $\frac{\partial \widehat{\theta}}{\partial \mu} < 0$ apply when δ is small enough. In fact, by differentiating on both sides of (13a), we can express $\frac{\partial \widehat{\theta}}{\partial \theta^*}$ and $\frac{\partial \widehat{\theta}}{\partial \mu}$ as a continuous function of δ, so there exists an $r > 0$ such that the signs of $\frac{\partial \widehat{\theta}}{\partial \theta^*}$ and $\frac{\partial \widehat{\theta}}{\partial \mu}$ do not change on neighborhood $B(\delta = 0, r)$.

Next, we analyze (13b). Denote $H(\frac{\theta^*-\widehat{\theta}}{\delta})$ by y in (13b). So (13b) can be written as

$$I = \frac{(1-c)X - R \cdot F \cdot (1-y)}{F \cdot y - c} \pi(\widehat{\theta}) \tag{A5}$$

Clearly, in (A5), $\frac{\partial I}{\partial \widehat{\theta}} > 0$. We can also prove that $\frac{\partial I}{\partial y} < 0$. In fact,

$$\begin{aligned}
\frac{\partial I}{\partial y} &= \frac{RF(Fy-c) - F[(1-c)X - R \cdot F \cdot (1-y)]}{(F \cdot y - c)^2} \pi(\widehat{\theta}) \\
&= F\frac{[FR - (1-c)X] - Rc}{(F \cdot y - c)^2} \pi(\widehat{\theta}) \\
&< 0
\end{aligned}$$

The inequality above is due to (10).

Finally, based on the analysis on (13a) and (13b), we can show the overall comparative statics: $\frac{\partial I}{\partial \theta^*} > 0$ and $\frac{\partial I}{\partial \mu} < 0$. The first is about $\frac{\partial I}{\partial \theta^*} > 0$. An increase in θ^* leads to $\widehat{\theta}$ increasing and y decreasing because $\frac{\partial \widehat{\theta}}{\partial \theta^*} > 1$ in (13a), which form two channels of forces in (A5) that push up I. The second is about $\frac{\partial I}{\partial \mu} < 0$. A decrease in μ leads to $\widehat{\theta}$ increasing in (13a), so in (A5) not only $\widehat{\theta}$ increases but also y decreases; both forces push up I.

Proof of Proposition 3:

We write the solution in (12a)-(12b) as the reaction function $\theta^*(I; \mu)$, where μ is the mean of public information. We have shown that $\frac{\partial \theta^*}{\partial I} > 0$ and it is easy to show $\frac{\partial \theta^*}{\partial \mu} < 0$. We write the

solution in (13a)-(13b) as the reaction function $I(\theta^*; u)$ where μ is the mean of the asset quality distribution. We have shown that $\frac{\partial I}{\partial \theta^*} > 0$ and $\frac{\partial I}{\partial \mu} < 0$. Therefore, a negative shock of μ lead to θ^* and I spiraling upward. Certainly, $\widehat{\theta}_{N\varepsilon} > \widehat{\theta}_N$, $\theta^*_{N\varepsilon} > \theta^*_N$ and $I_{N\varepsilon} > I_N$.

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